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Statistical Treatment of Turbulent Polydisperse Particle Systems

A Non-sectional PDF Approach

 Springer

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To my wife Parvaneh for her patience and support

– Sina –

Preface

This monograph is an advanced text in fluid dynamics intended for researchers and graduate students in the field of multiphase flows or powder technology. In this monograph, a novel formulation strategy, based on the probability density function, for general multiphase flow systems is introduced and thoroughly discussed. The novelty of the current approach, compared to the other non-PDF-based methods, lies in the fact that formulation starts by writing the fundamental ordinary differential equations (ODEs) governing the motion of Lagrangian fluid and discrete particles. This allows physical models to be applied at that level.

The other objective is to apply the aforementioned strategy to the formulation of turbulent particle-flow systems without assuming a constant particle size. Although there have been several attempts to include the particle size distribution into the governing equations none of these previous formulations can coherently incorporate the size distribution into the governing equations. However, it is shown in this monograph how particle size can be added naturally to the equations by first writing the fundamental ODEs governing the motion of the individual fluid or discrete particles.

In the first chapter of the book, we first introduce the multiphase flow phenomena and different flow regimes encountered in a typical two-phase flow system and introduce the simulation and modelling methods applicable to each regime. The outline of the book will also become clear in the introduction.

In Chap. 3, probability theory will be discussed and both single variable and multivariate probability density functions will be defined with some useful standard PDFs and their properties that will be used in the later chapters. Then a stochastic process will be defined using physical examples and numerical simulation to clarify the concept. Standard processes are introduced and their properties are examined using numerical simulations and analytical proofs. Readers already familiar with these concepts can safely skip this chapter and start at Chap. 3.

Kinetic and granular theory will be discussed to introduce the concept of the PDF transport equations and averaging process. This problem holds many resemblances to our particle-fluid system and the physical interpretation of many mathematical definitions will become clear in this chapter. Then we turn to the main problem and give the mathematical description of the fluid-particle system using both Eulerian and Lagrangian approaches and define the Mass Density Function, which is usually used to write the equations instead of a PDF due to its

intuitive physical interpretation. The state vector of the system will be defined and transition from a deterministic definition to a stochastic definition, its necessity for a turbulent system and its consequence will be discussed and the derivation of the general transport equation will be detailed.

Using the general transport equations, the field equations are derived and different terms will be explained and interpreted physically. Application of the method to mono-sized and poly-sized particles will be introduced, and the consequences of adding a length scale to the state vector due to poly-dispersity of the flow will become clear. Finally, the closure problem will be discussed and several applicable methods will be introduced and suggestions will be provided.

Southampton, March 2014

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Acronyms

BBGKY	Bogoliubov Born Green Kirkwood Yvon
CK	Chapman–Kolmogorov
DFMM	Direct Fractional Method of Moments
DNS	Direct Numerical Simulation
EE	Eulerian–Eulerian
EL	Eulerian–Lagrangian
GL	Grunwald–Letnikov
KTGF	Kinetic Theory of Granular Flow
KTGK	Kinetic Theory of Gases
LES	Large Eddy Simulation
LPM	Laguerre Polynomial Method
MDF	Mass Density Function
MEM	Maximum Entropy Method
MGF	Moment Generating Function
NS	Navier–Stokes
ODE	Ordinary Differential Equation
PDF	Probability Density Function
RANS	Reynolds Average Navier–Stokes
RL	Riemann–Liouville
SDE	Stochastic Differential Equation