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Robust and Optimal Control

A Two-port Framework Approach

 Springer

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Series Editors' Foreword

The series *Advances in Industrial Control* aims to report and encourage technology transfer in control engineering. The rapid development of control technology has an impact on all areas of the control discipline: new theory, new controllers, actuators, sensors, new industrial processes, computer methods, new applications, new philosophies . . . , new challenges. Much of this development work resides in industrial reports, feasibility study papers, and the reports of advanced collaborative projects. The series offers an opportunity for researchers to present an extended exposition of such new work in all aspects of industrial control for wider and rapid dissemination.

The *Advances in Industrial Control* monograph series started in 1992, and in many ways the sequence of volumes in the series provides an insight into what industries and what control system techniques were the focus of attention over the years. A look at the series titles on *robust* control yields the following list:

- *Robust Multivariable Flight Control* by Richard J. Adams, James M. Buffington, Andrew G. Sparks, and Siva S. Banda (ISBN 978-3-540-19906-9, 1994)
- *H_∞ Aerospace Control Design* by Richard A. Hyde (ISBN 978-3-540-19960-1, 1995)
- *Robust Estimation and Failure Detection* by Rami S. Mangoubi (ISBN 978-3-540-76251-5, 1998)
- *Robust Aeroservoelastic Stability Analysis* by Rick Lind and Marty Brenner (ISBN 978-1-85233-096-5, 1999)
- *Robust Control of Diesel Ship Propulsion* by Nikolaos Xiros (ISBN 978-1-85233-543-4, 2002)
- *Robust Autonomous Guidance* by Alberto Isidori, Lorenzo Marconi, and Andrea Serrani (ISBN 978-1-85233-695-0, 2003)
- *Nonlinear H_2/H_∞ Constrained Feedback Control* by Murad Abu-Khalaf, Jie Huang, and Frank L. Lewis (ISBN 978-1-84628-349-9, 2006)
- *Structured Controllers for Uncertain Systems* by Rosario Toscano (ISBN 978-1-4471-5187-6, 2013)

And from the sister series, *Advanced Textbooks in Control and Signal Processing* come:

- *Robust Control Design with MATLAB*[®] by Da-Wei Gu, Petko Hr. Petrov, and Mihail M. Konstantinov (2nd edition ISBN 978-1-4471-4681-0, 2013)
- *Robust and Adaptive Control* by Eugene Lavretsky and Kevin Wise (ISBN 978-1-4471-4395-6, 2013)

Clearly, robust control has seen a steady stream of monographs and books in both series. There is no doubt that the work of George Zames, Bruce Francis, John Doyle, Keith Glover, and many others created a paradigm change in control systems theory. Also note the number of aerospace-industry applications in the above list of texts. This emphasis can be ascribed to the availability of accurate high-dimensional multivariable models of aerospace systems within the industry, to the wide range of operating envelopes and therefore models, that aerospace vehicles traverse during a flight and the facility of optimization-based robust-control techniques in dealing with multivariable systems and their operational constraints.

From time to time, the *Advances in Industrial Control* series publishes a monograph that is theoretical and tutorial in content. This contrasts with most entries to the series that contain a mix of the theoretical, the practical, and the industrial. This monograph *Robust and Optimal Control* by Mi-Ching Tsai and Da-Wei Gu is one of those exceptions. The authors themselves actually raise the question “Why another book on the topic of Robust Control?” and their answer is that they have devised a new route to understanding the derivations and computation of robust and optimal controllers that they believe is a valuable addition to the literature of the subject. Their two-port approach is claimed to be more accessible to an engineering readership and to resonate in particular with an electrical- and electronic-engineering readership. The theoretical developments reported in the monograph are fully supported by detailed chapters covering all the background material and MATLAB[®] code and illustrated in a simple but persuasive servo-motor-control problem in the final chapter of the monograph. The list of monographs and textbooks on robust control shows that there is a continuing industrial interest in this field and for this reason this monograph is a valuable entry to the *Advances in Industrial Control* monograph series.

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Preface

This is a book on robust and optimal control of linear, time-invariant systems.

The human being always seeks better results in all activities, and this desire pushes the advance of science and technology. This happens in the control systems and control engineering area as well. It is desirable to develop a control strategy, a controller, for a dynamic system under consideration, to satisfy all possible constraints and to optimize a certain cost function which reflects the design objectives. This is so-called an optimal control problem. Such problems can be traced back to as early as the seventeenth century, in the Brachistochrone curve problem raised by Johann Bernoulli. Solution approaches towards such problems include the classical Calculus of Variations, Pontryagin's Maximum Principle, Dynamic Programming, (Differential) Game Theory, and Nonsmooth Optimization. These procedures are complicated, and solutions do not always exist. Fortunately, for linear time-invariant (LTI) systems, many cases would have satisfactory results, and this book presents a powerful approach which is also easy to understand, for electrical engineers in particular, we hope.

On the other hand, robustness is another vitally important issue in control systems design. A successfully designed automatic control system should be always able to maintain stability and an acceptable performance level despite uncertainties in system dynamics and/or in the operation environment to a certain degree, while such uncertainties inevitably exist in any real-world control system. In the late 1970s and early 1980s with the pioneering work by Zames [8] and Zames and Francis [9], a theoretic development, now known as the H_∞ optimal control theory, was taking shape. Robust controllers for LTI systems can be found by solving corresponding optimization problems. Robustness is thus achieved by designing a controller which attains certain optimality of the closed-loop system. The H_∞ and related optimization approaches are well developed and elegant. They provide systematic design procedures, in particular, for multi-input, multi-output linear systems.

There have been a number of books on this subject. Some books are on the underlying theories and the derivation of solution formulae [1, 3, 5, 7, 10]. Others are more on design methodologies, application of such theories, and implementation software [2, 6]. Naturally, a question arises, "Do we need another book on this subject?"

It seems satisfactory that practicing control engineers can use available solution formulae and software routines to work out robust and optimal controllers for given design problems, when they know well the underlying control systems and design specifications. However, are we happy with such designed controllers without knowing exactly how the formulae are derived and on what grounds the solution procedures are based? As control engineers, are we confident enough to implement such designed controllers? Answers to above queries might be obvious, and there are sources for us to know the theories of design approaches, as pointed out earlier. The problem is that the theory behind the state-space approaches presented in [10] and other books is very mathematically oriented and difficult for engineers, and students as well, to understand. Hence, is it possible to present the robust and optimal control theory for LTI systems in a way such that engineers and students can follow and grasp the essence of the solution approach? This motivated the research and writing of the present book.

This book presents an alternative approach to find a robust controller via optimization. This approach is based on the chain scattering decomposition (CSD), initiated by Professor Hidenori Kimura [4] and references therein who also named this as chain scattering description. CSD uses the configuration of two-port circuits which is a fundamental ingredient of electrical engineering and is familiar to all electrical engineers and students with basic electrical engineering knowledge. It is shown in the book that (sub)optimal H_∞ , H_2 as well as stabilizing controllers can be synthesized following the CSD approach. The book starts from the well-known linear fractional transformation (LFT), in which a control design problem can easily be formulated, and then converts LFT into CSD format. From the CSD formulation, the desired controller can be directly derived by using the framework proposed in the book in an intuitive and convenient way. The results are complete and valid for general system settings. The derivation of solution formulae is straightforward and uses no mathematics beyond linear algebra. It is hoped that readers may obtain insight from this robust and optimal controller synthesis approach, rather being bewildered in a mathematics maze.

The prerequisites for reading this book are classical control and state variable control courses at undergraduate level as well as elementary knowledge of linear algebra and electrical circuits. This book is intended to be used as a textbook for an introductory graduate course or for senior undergraduate course. It is also our intention to prepare this book for control engineers training courses on robust and optimal control systems design for linear time-invariant systems. With the above consideration in mind, we use plenty of simple yet illustrative worked examples throughout the book to help readers to understand the concepts and to see how the theory develops. Where appropriate, *MATLAB* codes for the examples are also included for readers to verify the results and to try on their own problems. Most chapters are followed with exercises for readers to digest the contents covered in the chapter. To further demonstrate the proposed approaches, in the last chapter, an application case study is presented which shows wider usage of the framework.

References

1. Green M, Limebeer DJN (1995) Linear robust control. Prentice Hall, Englewood Cliffs
2. Gu DW, Petkow PH, Konstantinov MM (2005) Robust control design with Matlab. Springer, London
3. Helton JW, Merino O (1998) Classical control using H_∞ methods-theory, optimization, and design. Society for industrial and Applied Mathematics, Baltimore
4. Kimura H (1997) Chain-scattering approach to H_∞ control. Birkhäuser, Boston
5. Maciejowski JM (1989) Multivariable feedback design. Addison-Wesley, Wokingham/Berkshire
6. Skogestad S, Postlethwaite I (2005) Multivariable feedback control: analysis and design. Wiley, New York
7. Stoorvogel AA (1992) The H_∞ control problem: a state space approach. Prentice Hall, Englewood Cliffs
8. Zames G (1981) Feedback and optimal sensitivity: model reference transformation, multiplicative seminorms, and approximated inverse. IEEE Trans Autom Control 26:301–320
9. Zames G, Francis BA (1983) Feedback, minimax sensitivity, and optimal robustness. IEEE Trans Autom Control 28:585–601
10. Zhou K, Doyle JC, Glover K (1996) Robust and optimal control. Prentice Hall, Upper Saddle River

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