

# Communications and Control Engineering

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# Cooperative Control of Multi-Agent Systems

Optimal and Adaptive Design Approaches

 Springer

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ISBN 978-1-4471-5573-7      ISBN 978-1-4471-5574-4 (eBook)  
DOI 10.1007/978-1-4471-5574-4  
Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2013950494

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Printed on acid-free paper

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*To Galina, for her beauty and her wonderful  
frame of mind*

Frank Lewis

*To Lin and my parents, for their uncondi-  
tional love*

Hongwei Zhang

*To my family, for all the support, love and  
kindness*

Kristian Hengster-Movric

*To my mother, because of whom I evaded  
the dark side of me*

Abhijit Das

# Preface

This book studies cooperative control of multi-agent dynamical systems interconnected by a communication network topology. In cooperative control, each system is endowed with its own state variable and dynamics. A fundamental problem in multi-agent dynamical systems on networks is the design of distributed protocols that guarantee consensus or synchronization in the sense that the states of all the systems reach the same value. The states could represent vehicle headings or positions, estimates of sensor readings in a sensor network, oscillation frequencies, trust opinions of each agent, and so on. In multi-agent systems, all systems should agree on the values of these quantities to achieve synchronized behavior.

Of fundamental concern for networked cooperative dynamical systems is the study of their interactions and collective behaviors under the influence of the information flow allowed in the communication network. This communication network can be modeled as a graph with directed edges or links corresponding to the allowed flow of information between the systems. The systems are modeled as the nodes in the graph and are sometimes called agents. Information in communication networks only travels directly between immediate neighbors in a graph. Nevertheless, if a graph is connected, then this locally transmitted information travels ultimately to every agent in the graph.

In cooperative control systems on graphs, there are intriguing interactions between the individual agent dynamics and the topology of the communication graph. The graph topology may severely limit the possible performance of any control laws used by the agents. Specifically, in cooperative control on graphs, all the control protocols must be *distributed* in the sense that the control law of each agent is only allowed to depend on information from its immediate neighbors in the graph topology. If enough care is not taken while designing the local agent control laws, the individual agent dynamics may be stable, but the networked systems on the graph may exhibit undesirable behaviors. Since the communication restrictions imposed by graph topologies can severely complicate the design of synchronization controllers, complex and intriguing behaviors are observed in multi-agent systems on graphs that do not occur in single-agent, centralized, or decentralized feedback control systems.

The study of networks of coupled dynamical systems arises in many fields of research. Charles Darwin showed that the interactions between coupled biological species over long time scales are responsible for natural selection. Adam Smith showed that the dynamical relationships between geopolitical entities are responsible for the balances in international finance and the wealth of nations. Distributed networks of coupled dynamical systems have received much attention over the years because they occur in many different fields including biological and social systems, physics and chemistry, and computer science. Various terms are used in literature for phenomena related to the collective behavior on networks of systems, such as flocking, consensus, synchronization, frequency matching, formation, rendezvous, and so on. Collective synchronization phenomena occur in biology, sociology, physics, chemistry, and human engineered systems. The nature of synchronization in different groups depends on the manner in which information is allowed to flow between the individuals of the group.

The collective motions of animal social groups are among the most beautiful sights in nature. Each individual has its own inclinations and motions, yet the aggregate motion makes the group appear to be a single entity with its own laws of motion, psychology, and responses to external events. Flocks of birds, herds of animals, and schools of fish are aggregate entities that take on an existence of their own due to the collective motion instincts of their individual members. Collective motions allow the group to achieve what the individual cannot. Collective synchronized motion is a product not of planned scripts, but of instantaneous decisions and responses by individual members.

Analysis of groups based on social behaviors is complex, yet the individuals in collectives appear to follow simple rules. In many biological and sociological groups such as schools of fish, bird flocks, mammal herds on the move, and human panic behavior in emergency building evacuation, evidence supports the idea that the decisions made by all the individuals follow simple local protocols based on their nearest neighbors. The collective motion of large groups can be captured by using a few simple rules governing the behavior of the individuals. These rules depend on the awareness of each individual of its neighbors.

Mechanisms of information transfer in groups involve questions such as how information about required motion directions, originally held by only a few informed individuals, can propagate through an entire group by simple mechanisms that are the same for every individual. The information flow between members of a social group is instrumental in determining the characteristics of the combined motion of the overall group.

The engineering study of multi-agent cooperative control systems uses principles observed in sociology, chemistry, and physics to obtain synchronized behavior of all systems by using simple local distributed control protocols that are the same for each agent and only depend on that agent's neighbors in the group. Applications have been to oscillator synchronization, aircraft formations, mobile sensor area coverage, spacecraft attitude alignment, vehicle routing in traffic systems, containment control of moving bodies, and biological cell sorting.

Optimal feedback control design has been responsible for much of the successful performance of engineered systems in aerospace, manufacturing, industrial processes, vehicles, ships, robotics, and elsewhere since the 1960s. Optimal control designs generally require complete information of the system dynamics and rely on off-line solutions of matrix design equations. Adaptive control is a powerful method for the design of dynamic controllers that are tuned online in real time to learn stabilizing feedback controllers for systems with unknown dynamics. Many successful applications have been made in manufacturing and aerospace systems, and elsewhere.

In this book, we use distributed cooperative control principles to design optimal control systems and adaptive control systems for multi-agent dynamics on graphs. These designs are complicated by the fact that all control protocols and parameter-tuning protocols must be distributed in the sense that they depend only on immediate neighbors in the graph. Optimal control for cooperative multi-agent systems is discussed in Part I of the book. Cooperative adaptive control is discussed in Part II.

Chapter 1 of this book presents an overview of synchronization behavior in nature and social systems. It is seen that distributed decisions made by each agent in a group based only on the information locally available to it can result in collective synchronized motion of the overall group. The idea of a communication graph that models the information flows in a multi-agent group is introduced. Synchronization and collective behavior phenomena are discussed in biological systems, physics and chemistry, and engineered systems. Various different graph topologies are presented including random graphs, small world networks, scale-free networks, and distance formation graphs. The early work in cooperative control systems on graphs is outlined.

Chapter 2 introduces cooperative synchronization control of multi-agent dynamical systems interconnected by a fixed communication graph topology. Each agent or node is mathematically modeled by a dynamical linear time-invariant system. A review is given of graph basics and algebraic graph theory, which studies certain matrices associated with the graph. Dynamical systems on graphs are introduced. The idea of distributed control and the consensus problem are introduced. We begin our study with first-order integrator dynamics for continuous-time systems and discrete-time systems. Then, results are given for second-order position-velocity systems which include motion control in formations. We present some key matrix analysis methods for systems on graphs that are important for the analysis and design of cooperative controllers.

In Part I of the book, which contains Chaps. 3–6, we study local and global optimal control for cooperative multi-agent systems linked to each other by a communication graph. In cooperative control systems on graphs it turns out that local optimality for each agent and global optimality for all the agents are not the same. The relations between stability and optimality are well understood for single-agent systems. However, there are more intriguing relations between stability and optimality in cooperative control than which appear in the single-agent case, since local stability and global team stability are not the same, and local agent optimality and global team optimality are not the same. New phenomena appear that are not

present for single-agent systems. Moreover, throughout everything the synchronization of the states of all agents must be guaranteed.

In Chap. 3, we study optimal control for continuous-time systems, and we shall see that local optimal design at each agent guarantees global synchronization of all agents to the same state values on any suitably connected digraph. Chapter 4 considers discrete-time systems and shows that an extra condition relating the local agent dynamics and the graph topology must be satisfied to guarantee global synchronization using local optimal design. Global optimization of collective group motions is more difficult than local optimization of the motion of each agent. A common problem in optimal decentralized control is that global optimization problems generally require global information from all the agents, which is not available to distributed controllers which can only use information from nearest neighbors. In Chap. 5, we shall see that globally optimal controls of distributed form may not exist on a given graph. To obtain globally optimal performance using distributed protocols that only depend on local agent information in the graph, the global performance index must be selected to depend on the graph properties in a certain way, specifically, through the graph Laplacian matrix. In Chap. 6, we define a different sort of global optimality for which distributed control solutions always exist on suitably connected graphs. There, we study multi-agent graphical games and show that if each agent optimizes its own local performance index, a Nash equilibrium is obtained.

In Part II of the book, which contains Chaps. 7–10, we show how to design cooperative adaptive controllers for multi-agent systems on graphs. These controllers allow synchronization of nonlinear systems where the agents have different dynamics. The dynamics do not need to be known and may have unknown disturbances. In adaptive controllers that are admissible for a prescribed communication graph topology, only distributed control protocols and distributed adaptive tuning laws are permitted. It is not straightforward to develop distributed adaptive tuning laws for cooperative agents on graphs that only require information from each agent and its neighbors. We show that the key to this is selecting special Lyapunov functions for adaptive control design that depend in specific ways on the graph topology. Such Lyapunov functions can be constructed using the concept of graph Laplacian potential, which depends on the communication graph topology.

In Chap. 7, we show that for networked multi-agent systems, there is an energy-like function, called the graph Laplacian potential, that depends on the communication graph topology. The Laplacian potential captures the notion of a virtual potential energy stored in the graph. The Laplacian potential is further used to construct Lyapunov functions that are suitable for the stability analysis of cooperative control systems on graphs. These Lyapunov functions depend on the graph topology, and based on them a Lyapunov analysis technique is introduced for cooperative multi-agent systems on graphs. Control protocols coming from such Lyapunov functions are distributed in form, depending only on information about the agent and its neighbors.

Chapter 8 covers cooperative adaptive control for systems with first-order nonlinear dynamics. The dynamics of the agents can be different, and they may be affected by disturbances. The dynamics may be unknown. A special Lyapunov func-



tion that depends on the graph topology is used to construct cooperative adaptive controllers wherein both the control protocols and the parameter tuning laws are distributed in the sense that they depend only on information available locally from the neighbors of each agent. Chapter 9 shows how to design adaptive controllers for nonlinear second-order multi-agent systems with position-velocity dynamics. Chapter 10 designs cooperative adaptive controllers for higher-order nonlinear systems with unknown dynamics and disturbances. The challenge of ensuring that both the control protocols and parameter tuning laws are distributed on the graph is confronted by using a Lyapunov function that involves two terms, one depending on the system dynamics and one depending on the communication graph topology.

# Acknowledgements

This work has been supported over the years by the U.S. National Science Foundation, the U.S. Army Research Office and TARDEC, the U.S. Air Force Office of Scientific Research, and the Nanjing University of Science & Technology Visiting Scholar Program. Recent support has been given by the National Natural Science Foundation of China (through the Academy of Mathematics and Systems Sciences, Chinese Academy of Sciences) and China Education Ministry Project 111 (through Northeastern University, Shenyang), the National Natural Science Foundation of China under grant 61304166, the Research Fund for the Doctoral Program of Higher Education under grant 20130184120013, and the China Fundamental Research Funds for the Central Universities under grants 2682013CX016 and SWJ-TU11ZT06.

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