Part I
Local Optimal Design for Cooperative Control in Multi-Agent Systems on Graphs

Cooperative control studies the dynamics of multi-agent dynamical systems linked to each other by a communication graph. The objective of cooperative control is to devise control protocols for the individual agents that guarantee synchronized behavior of the states of all the agents in some prescribed sense. In cooperative systems, any control protocol must be distributed in the sense that it respects the prescribed graph topology; that is, the control protocol for each agent is allowed to depend only on information about that agent and its neighbors in the graph.

In Part I of the book, we will study local and global optimal control for cooperative multi-agent systems linked to each other by a communication graph. In Chap. 3, we study continuous-time systems, and we shall see that local optimal design at each agent guarantees global synchronization of all agents to the same state values on any suitably connected digraph. Chapter 4 considers discrete-time systems and shows that an extra condition relating the local agent dynamics and the graph topology must be satisfied to guarantee global synchronization using local optimal design.

In cooperative control systems on graphs, it turns out that local optimality for each agent and global optimality for all the agents are not the same. The relations between stability and optimality are well understood for single-agent systems. However, there are more intriguing relations between stability and optimality in cooperative control than which appear in the single-agent case, since local stability and global team stability are not the same, and local agent optimality and global team optimality are not the same. New phenomena appear that are not present for single-agent systems. Moreover, throughout everything the synchronization of the states of all agents must be guaranteed.

A common problem in optimal decentralized control is that global optimization problems generally require global information from all the agents, which is not available to distributed controllers. In Chap. 5, we shall see that globally optimal controls of distributed form may not exist on a given graph. To obtain globally optimal performance using distributed protocols that only depend on local agent information in the graph, the global performance index must be selected to depend on the graph properties in a certain way, specifically, through the graph Laplacian matrix.

In Chap. 6, we will define a different sort of global optimality for which distributed control solutions always exist on suitably connected graphs. There, we study multi-agent graphical games and show that if each agent optimizes its own local performance index, a Nash equilibrium is obtained.