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# Linear Parameter-Varying Control for Engineering Applications

 Springer

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*To our patient, kind, and loving families*

# Preface

The objective of this brief is to carefully illustrate a procedure of applying linear parameter varying (LPV) control to a class of dynamic systems via a systematic synthesis of gain-scheduling controllers with guaranteed stability and performance. The existing LPV control theories rely on the use of either  $\mathcal{H}_\infty$  or  $\mathcal{H}_2$  norm to specify the performance of the LPV system. The challenge that arises with LPV control for engineers is twofold. First, there is no systematic procedure in applying existing LPV control system theory to solve practical engineering problems from modeling to control design. Second, there exists no LPV control synthesis theory to design LPV controllers with hard constraints. For example, physical systems usually have hard constraints on their required performance outputs along with their sensors and actuators. Furthermore, the  $\mathcal{H}_\infty$  and  $\mathcal{H}_2$  performance criteria cannot provide hard constraints on system outputs. As a result, engineers in industry could find it difficult to utilize the current LPV methods in practical applications. To address these challenges, in this brief, gain-scheduling control with engineering applications is covered in detail, including the LPV modeling, the control problem formulation, and the LPV system performance specification. In addition, a new performance specification is considered which is capable of providing LPV control design with hard constraints on system outputs. The LPV design and control synthesis procedures in this brief are illustrated through an engine air-to-fuel ratio control system, an engine variable valve timing control system, and an LPV control design example with hard constraints. After reading this brief, the reader will be able to apply a collection of LPV control synthesis techniques to design gain-scheduling controllers for their own engineering applications. This brief provides detailed step-by-step LPV modeling and control design strategies along with a new performance specification so that engineers can apply state-of-the-art LPV control synthesis to solve their own engineering problems. In addition, this brief should serve as a bridge between the  $\mathcal{H}_\infty$  and  $\mathcal{H}_2$  control theory and the real-world application of gain-scheduling control.

The material presented in this brief is the result of research performed to develop gain-scheduling controllers using LPV control theory. Our goal at the beginning of this research was to develop a systematic procedure for designing gain-scheduling controllers. Since we started working in this area, we have written numerous journal and conference publications to disseminate our work.

Specifically, material from the journal papers [60–62] make up a large portion of this brief. In addition to the material from these three journal papers, we have also included a portion of our recent research on designing gain-scheduling controllers that can provide hard constraints on system outputs.

The intended audience of this brief are control engineers who are interested in designing gain-scheduling controllers for practical problems. The examples included in this brief will provide them with insight and guidance when designing gain-scheduling controllers using LPV methods for their practical problems. Control research engineers are also expected to be able to use this brief. Finally, this brief is also capable of being used as a teaching supplement to introduce graduate students with a prerequisite understanding of robust control to the area of LPV control.

We would like to acknowledge our co-authors Dr. Ryoza Nagamune and Dr. Zhen Ren for their contributions to the papers they helped us publish. Specifically, we would like to thank Dr. Ryoza Nagamune, from the University of British Columbia, for his valuable contributions to our paper “Gain-Scheduling Control of Port-Fuel-Injection Processes” during the revision process. We would also like to thank Dr. Zhen Ren for his hard work developing and building the test bench for the variable valve timing actuator. Additionally, we would also like to thank Dr. Xiaojian Yang for his work developing the mixed mean value and crank-based engine model used to validate the gain-scheduling controller developed in [Chap. 4](#) of this brief.

East Lansing, MI, January 2013

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# Acronyms

A/F	Air-to-fuel ratio
BMEP	Brake mean effective pressure
CAN	Controller area network
DI	Direct injection
ECU	Engine control unit
EGR	Exhaust gas recirculation
HIL	Hardware-in-the-loop
IC	Internal combustion
LFT	Linear fractional transformation
LPV	Linear parameter varying
LMI	Linear matrix inequality
LTI	Linear time invariant
LTV	Linear time varying
MAP	Manifold air pressure
OCC	Output covariance constraint
PFI	Port fuel injection
PI	Proportional-integral
PID	Proportional-integral-derivative
PWM	Pulse width modulation
SISO	Single-input single-output
UEGO	Universal exhaust gas oxygen
VVT	Variable valve timing