Advances in the Complex Variable Boundary Element Method

T.V. Hromadka II and R.J. Whitley

Advances in the Complex Variable Boundary Element Method

With 157 Figures

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Dedication

For my father.

Preface

Since its inception by Hromadka and Guymon in 1983, the Complex Variable Boundary Element Method or CVBEM has been the subject of several theoretical adventures as well as numerous exciting applications. The CVBEM is a numerical application of the Cauchy Integral theorem (well-known to students of complex variables) to two-dimensional potential problems involving the Laplace or Poisson equations. Because the numerical application is analytic, the approximation *exactly* solves the Laplace equation. This attribute of the CVBEM is a distinct advantage over other numerical techniques that develop only an inexact approximation of the Laplace equation.

In this book, several of the advances in CVBEM technology, that have evolved since 1983, are assembled according to primary topics including theoretical developments, applications, and CVBEM modeling error analysis. The book is self-contained on a chapter basis so that the reader can go to the chapter of interest rather than necessarily reading the entire prior material. Most of the applications presented in this book are based on the computer programs listed in the prior CVBEM book published by Springer-Verlag (Hromadka and Lai, 1987) and so are not republished here.

The book also contains course notes prepared for graduate level projects conducted at California State University, Fullerton, Department of Mathematics, in the University's Applied Mathematics Masters Degree program. The book's material provides a novel application of complex variables and analytic functions for a senior level or graduate level course in complex variables, as well as an applied mathematics course in engineering analysis.

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In preparing any book that involves complex mathematics, the task of proofing the work and including revisions and comments is truly a difficult one.

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