Model Predictive Vibration Control

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Efficient Constrained MPC Vibration Control for Lightly Damped Mechanical Structures

Springer

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"Prediction is very difficult,
especially about the future."

Niels Bohr

This book is dedicated to our dearest better halves Janka Sipos and Marta Rohal'ová-Ilkivová whose passion, love and support fuels our lives; to our parents Erika and András Takács, Ol'ga and the memory of Ivan Rohal'-Ilkiv who have raised us with respect for knowledge, wisdom and science; and to our students who unlike our family and loved ones will have to read the rest of the book too…

Preface

This book provides insight into the model predictive control of lightly damped vibrating structures. Conclusions of ongoing research in the field, up-to-date experimental results and the doctoral dissertation thesis titled “Efficient Model Predictive Control Applied on Active Vibration Attenuation” by Gergely Takács have been summarized into a clearly presented and accessible form. The book is intended for use in undergraduate or graduate level university curricula or for industrial practitioners interested in computationally efficient predictive control utilized in active vibration attenuation. It is assumed the reader has a very basic understanding of linear control theory and vibration mechanics.

The control strategy discussed in this book is based on the idea of using a mathematical model to predict the future behavior of a vibrating system and selecting the best control moves based on an optimization procedure using this predicted information. This method is known as model predictive control (MPC) and due to its intense computational requirements has been so far used mainly to control processes with very slow dynamics. The control moves computed by MPC will not only be ideal in a sense of damping performance, but they will also respect process constraints arising from physical actuator limitations, safety or economic reasons. This title will introduce the current state and the theoretical particulars behind this advanced control strategy and show how it can be implemented using piezoelectric actuators to lightly damped vibrating structures, in order to eliminate or attenuate undesired vibrations.

Using more than 170 illustrations, photographs, diagrams and several tables, the book will take the reader through the necessary steps in understanding the fundamentals of active vibration control (AVC), give a thorough review of the current state of model predictive control and finally will also introduce the implementation of computationally efficient MPC algorithms and compare different predictive control strategies in simulation and experiment.

Both active vibration attenuation and model predictive control have been treated in numerous excellent books already. So why would we need another publication on these topics? Works discussing the field of (active) vibration control are generally limited to presenting traditional control methods ranging from positive position feedback (PPF) to linear quadratic control (LQ). The progress of control theory has not stopped with providing us the tools to synthesize and implement simple controllers such as proportional integrating derivative (PID). Modern optimization-based control methods, such as model predictive control are generally not considered for active vibration control applications. This can be partly attributed to the fact that the results of control theory tend to be transferred to real-life applications very slowly. The other major reason is due to the obvious implementation limitations: the sampling speeds usually encountered in AVC are too fast for real-time deployment. Advantages of predictive controllers over traditional controllers are not limited to an increased performance, but these methods also handle process, actuator and safety-related constraints on an algorithmic level.

On the other hand, either the books published on the topic of model predictive control are focused exclusively on the theory in deep mathematical detail or, even if practical implementation examples are given, they are limited to processes with slow dynamics. The reason for this is that implementation of predictive controllers on petrochemical plants, heaters and other slow processes do not invoke the computational time issue. Applying predictive control in active vibration attenuation is therefore not a topic of these publications. This book is distinct from general works on AVC or MPC because it presents the multi-disciplinary area of predictive control applied in vibration control, treating the subject as one compound problem. We offer a specific cross-section of these two actual and attractive engineering fields and suggest solutions for the research and industrial community.

Gergely Takács is currently a research engineer at the Institute of Automation, Measurement and Applied Informatics of the Faculty of Mechanical Engineering of the Slovak University of Technology in Bratislava. He has received his PhD degree in Mechatronics from the Slovak University of Technology in 2009. His recent doctoral studies and commencing academic career have been fully devoted to the application of computationally efficient model predictive controllers in the active vibration control of lightly damped structures. His research interests include active vibration control, smart materials, advanced actuators, and computationally efficient model predictive control. Gergely Takács is a member of IEEE.

Boris Rohal'-Ilkiv has received his degrees from the Slovak University of Technology in Bratislava, the Faculty of Mechanical Engineering, in control engineering. Currently, he is a tenured professor at the Institute of Automation, Measurement and Applied Informatics at the Faculty of Mechanical Engineering, where he is an active lecturer and researcher in the area of dynamical systems modeling and control. He has devoted the majority of his academic career to model predictive control, with a special attention focused at practical real-time controller implementation issues. Boris Rohal'-Ilkiv is a member of IEEE.

Bratislava, August 2011

Gergely Takács
Boris Rohal'-Ilkiv

Contact

Feel free to contact us with any matter related to this book or its general topic. Let us know what you think, if you have found mistakes, missing references or anything else. Your feedback is highly appreciated and we will do our best to rectify any insufficiencies for further editions. Unsolicited grant proposals and agencies are welcome as well:

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Neither of us is a native speaker of the English language, so we have passed on the manuscript to a third person who has learned English as a third language. But definitely better than we did. So our thanks goes to Janka Sipos, who despite being an English Literature major has chewed through the highly technical manuscript and corrected our spelling and grammatical mistakes. Any typos, spelling and grammatical errors left here are intentional and congratulations to those who have found them. No, seriously, please let us know.

During the preparation of the book manuscript, we have been continually seeking the rightful owners of those photographs and illustrations we have found in other sources and decided to include here. We would like to express our gratitude to the individuals and institutions who took the time to reply to our requests and have explicitly given permission to use these materials; including the United States National Aeronautics and Space Administration (NASA), the European Space Agency (ESA), The Boeing Company, Liebherr-International Deutschland

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Dunajská Streda, Slovakia, July 2011 Gergely Takács

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Bratislava, Slovakia, June 2011 Boris Rohal'-Ilkiv

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SLOVAK RESEARCH
AND DEVELOPMENT
AGENCY

Contents

Part I Vibration Control

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Abbreviations

AAMPC	Autonomous augmented MPC
A/D	Analog to digital
ADP	Ammonium dihydrogen phosphate
AFM	Atomic force microscope
AMB	Active magnetic bearing
ANC	Active noise control
ANN	Artificial neural network
ANSYS	Analysis system (software)
APB-IPM	Approximate primal-barrier interior-point method
APDL	ANSYS parametric design language
ARE	Algebraic Ricatti equation
AS	Active set
ATLAS	Automatically tuned linear algebra software (software)
ATW	Aerostructures test wing
AVA	Active vibration attenuation
AVC	Active vibration control
AVD	Active vibration damping
AVI	Active vibration insulation
BLAS	Basic linear algebra subprograms (software)
BNC	Bayonet Neil-Concelman connector
BNT	Bismuth sodium titanate
BST	Binary search tree
CAVF	Constant amplitude velocity feedback
CCD	Charge coupled device
CGVF	Constant gain velocity feedback
CPU	Central processing unit
CRHPC	Constrained receding horizon predictive control
D/A	Digital to analog
DAQ	Data acquisition
DARE	Discrete time algebraic Ricatti equation
DMC	Dynamic matrix control

DNA	Deoxyribonucleic acid
DOF	Degrees of freedom
ENRMPC	Extended Newton–Raphson MPC
EAP	Electroactive polymer
EGR	Exhaust gas recirculation
eMPC	Explicit MPC, see MPMPC
EP	Equality problem
ER	Electrorheological
ES	Electrostrictive
ESSP	Electro-statically stricted polymer
EVE	Electro-viscoelastic elastomer
FE	Finite element
FEA	Finite element analysis
FEM	Finite element method
FFT	Fast Fourier transformation
FIR	Finite impulse response
FPE	Final prediction error
FPGA	Field-programmable gate array
FSR	Finite step response
GA	Genetic algorithm
GPC	Generalized predictive control
HDA	High displacement actuator
HEB	European standard wide flange beam designation
HVAC	Heating, ventilation and air conditioning
HW	Hardware
I/O	Input–output
IP	Interior point
IPMC	Ionic polymer metallic composite
ISS	International space station
KKT	Karush–Kuhn–Tucker
LAPACK	Linear algebra package (software)
Laser	Light amplification by the stimulated emission of radiation
LDV	Laser Doppler vibrometer
LMI	Linear matrix inequality
LP	Linear programming
LQ	Linear quadratic
LQG	Linear quadratic Gaussian
LQR	Linear quadratic regulator
LTI	Linear time-invariant
MAC	Model algorithmic control
MAGLEV	Magnetic levitation
MDOF	Multiple degrees of freedom
MEMS	Micro-electromechanical systems
MFC	Macrofiber composite
MFM	Magnetic force microscope

MHO	Moving horizon observer
MIMO	Multiple-input, multiple-output
MP	Multi-parametric
MPC	Model predictive control
MPMPC	Multi-parametric model predictive control, also known as explicit MPC
MPT	Multi-parametric toolbox (software)
MR	Magnetorheological
MRI	Magnetic resonance imaging
MS	Magnetostrictive
NiTi	Nickel–titanium
NiTiNOL	Nickel–titanium NOL (alloy), nitinol
NMPC	Nonlinear model predictive control
NOL	Naval Ordnance Laboratory, see nitinol and terfenol
NP	Non-deterministic polynomial-time (hard)
NPF	Negative position feedback
NR	Newton–Raphson (procedure)
NRMPC	Newton–Raphson MPC
NRMPC_{k+1}	Original extended Newton–Raphson MPC (1 step)
NRMPC_{k+2}	Extended Newton–Raphson MPC (2 steps)
NRMPC_{k+3}	Extended Newton–Raphson MPC (3 steps)
OKID	Observer Kalman filter identification
ORM	Optimal region merging
PAF	Payload adapter fitting
PCI	Peripheral component interconnect
PF	Position feedback
PID	Proportional-integral-derivative
PPF	Positive position feedback
PLC	Programmable logic controller
PLZT	Lead lanthanum zirconate titanate
PMN	Lead magnesium niobate
PMN-PT	Lead magnesium niobate–lead titanate
PVF	Positive position feedback
PSD	Proportional-summing-derivative
PT	Lead titanate
PVF	Positive velocity feedback
PWA	Piecewise-affine
PWL	Piecewise-linear
PZT	Lead zirconate-titanate
PZT5A	A certain type of PZT material
QP	Quadratic programming
qpOASES	A type of online active set quadratic programming solver (software)
QPMPC	Quadratic programming MPC
RAM	Random access memory

RMS	Root mean square
RTW	Real-time workshop (software)
SDOF	Single degree of freedom
SDP	Semidefinite programming
SGPC	Stable generalized predictive control
SeDuMi	Self-dual-minimization (software)
SI	International system of units (Système international d'unités)
SIORHC	Stabilizing input/output receding horizon control
SISO	Single-input, single output
SMA	Shape memory alloy
SOCP	Second order cone programming
STOL	Short takeoff and landing
TCP/IP	Transmission control protocol / Internet protocol
TB	Tungsten bronze
TEFLON	Tetrafluorethylene
Terfenol-D	Terbium ferrum NOL dysprosium
TET	Task execution time
UAV	Unmanned aerial vehicle
UNF	Unified fine thread
USB	Universal serial bus
VTOL	Vertical takeoff and landing
xPC	xPC Target—rapid prototyping platform (software)
YALMIP	Yet another LMI parser (software)

Symbols

Greek Lower Case Letters

α	Rayleigh mass damping factor
α_l	Line search coefficient
α_s	Eigenvector matrix scaler
β	Rayleigh stiffness damping factor
γ	Prediction performance bound
γ_s	Shear strain
δ	Eigenvector
δ_d	Damping ratio (with physical dimension)
ε	Mechanical strain
ε_σ	Permittivity under constant or no stress
ε_S	Permittivity under constant strain
ζ	Dimensionless damping ratio
ζ_{fil}	Filter damping
η	Available degrees of freedom in the terminal state
η_v	Viscosity coefficient
θ	Angle in radians
θ_p	Active set coefficient vector
ι	Lagrange multiplier
ι_{ms}	Magnetostrictive coefficient
κ	Eigenvalue
λ	Lagrange multiplier
μ	Numerical vector scaler
μ_σ	Magnetic permeability at constant stress
ν	Lagrange multipliers for the terminal condition
ν_{xz}	Poisson's ratio in a given direction
ξ	Modal participation factor
ξ_{fil}	Diagonal matrix containing filter damping terms
π	Ratio of a circle's area to the square of its radius
ρ	Density

σ	Stress
σ_s	Singular value
τ	Shear stress
τ_d	Delay
τ_Y	Yield strength
υ	Simplification constant in distributed parameter systems
ϕ	Phase shift
χ	Root function defined by the NRMPC problem
ψ	Boundary function in interior point QP algorithms
ω_f	Angular frequency of a forced input
ω_{fil}	Frequency of a filter
ω_n	Angular natural frequency
ω_i	i-th angular natural frequency
ω_d	Damped angular natural frequency

Greek Capital Letters

Γ	Definition matrix of ellipsoids
Γ_f	f partition of $\Gamma_{\mathbf{Z}}^{-1}$
Γf_x	Mixed f, x partition of $\Gamma_{\mathbf{Z}}^{-1}$
Γ_x	x partition of $\Gamma_{\mathbf{Z}}^{-1}$
Γ_{xf}	Mixed x, f partition of $\Gamma_{\mathbf{Z}}^{-1}$
$\Gamma_{\mathbf{X}}$	Definition matrix of the invariant ellipsoid E_x in state-space, intersection of E_z with the state-space
$\Gamma_{\mathbf{XZ}}$	Definition matrix of the invariant ellipsoid E_{xz} created by the projection of the augmented invariant ellipsoid E_z
$\Gamma_{\mathbf{Z}}^{-1}$	Definition matrix of the augmented invariant ellipsoid E_{z}
$\bar{\Gamma}_z^{-1}$	One step ahead iteration of $\Gamma_{\mathbf{Z}}^{-1}$
$\tilde{\Gamma}_z^{-1}$	Two steps ahead iteration of $\Gamma_{\mathbf{Z}}^{-1}$
Δ	Eigenvector matrix
Θ	Simplification matrix in NRMPC
Θp	Active set coefficient matrix
Λ	Eigenvalue matrix
Λ_{fil}	Diagonal matrix of the squares of filter frequencies
Ξ	Boundary condition
Π	Target set
Π_{na}	Maximal invariant target set created by a finite constraint checking horizon
Π_s	Simplified invariant target set defined as a low complexity polytope
Π_∞	Maximal invariant target set
Σ	Transformation matrix in optimized dynamics NRMPC
Φ	Closed loop system matrix
Φ_{eq}	Equality constraint in general

Φ_{in}	Inequality constraint in general
Ψ	Closed loop matrix of the augmented system
Ω	Separation function

Latin Lower Case Letters

a_i	Elements of the active set
b	Viscous damping coefficient
b_c^i	i-th row of the constraint vector $\mathbf{b}_0$
c	Scalar perturbation, perturbation vector for $n_u > 1$
d	Element of the piezoelectric matrix
d_0	Initial deflection of the beam tip at $t = 0$ s
d_t	Deflection of the beam tip at time t
d_{ts}	Deflection boundary after which the state is the part of the target set
e	Scalar error, difference between reference and measured output
f	Generic function
f_0	First or dominant mechanical resonance frequency
g	Generic function
h	Scaler
i	i-th element in a series, index, generic iteration
k	Linear stiffness coefficient/discrete time step
l	Length
m	Point mass
n	Discrete dimension
n_b	Block adaptive update speed
n_c	Control horizon
n_{min}	Number of samples necessary to enter the target set from the given initial condition
n_p	Prediction horizon
n_u	Input dimension
n_x	State dimension
n_y	Output dimension
o_i	Perpendicular distance of the i-th hyperplane from the origin
p	Iteration of the quadratic programming algorithm
q	Displacement coordinate
r	Scalar input penalty/setpoint
s	Mechanical compliance in a given direction
t	Continuous time
t_{avg}	Average task execution time
t_{max}	Maximal task execution time
t_{min}	Minimal task execution time
t_{std}	Standard deviation of the average task execution time
u	Input

u_{max}	Maximal (peak) input
$\bar{u}$	Upper input constraint
$\underline{u}$	Lower input constraint
Δu	First difference of input
$\Delta\bar{u}$	Upper input rate constraint
$\Delta\underline{u}$	Lower input rate constraint
$u(s)$	Input in Laplace domain
$v(s)$	Disturbance in Laplace domain
$w(s)$	Reference in Laplace domain
x	State
$\bar{x}$	Upper state constraint
$\underline{x}$	Lower state constraint
$\tilde{x}$	Augmented state (integral action)
$\breve{x}$	Previous value of x, where $x = (\mathbf{A} + \mathbf{BK})\breve{x}$
Δx	First difference of state
$\Delta\bar{x}$	Upper state rate constraint
$\Delta\underline{x}$	Lower state rate constraint
x_0	Initial state, state x_k at time $k = 0$
y	Output
$\bar{y}$	Upper output constraint
$\underline{y}$	Lower output constraint
Δy	First difference of output
$\Delta\bar{y}$	Upper output rate constraint
$\Delta\underline{y}$	Lower output rate constraint
y_{avg}	Arithmetic mean of beam tip vibrations
y_{max}	Peak value of mean compensated beam tip vibrations
$y(s)$	Output in Laplace domain

Latin Capital Letters

A	Area
$\bar{A}$	Amplitude gain in constant amplitude velocity feedback
A_n	Integration constant in distributed parameter systems
A_c^i	i-th row of the constraint matrix $\mathbf{A}_c$
B_n	Integration constant in distributed parameter systems
E	Modulus of elasticity (Young's modulus)
E_x	Intersection of the augmented ellipsoid E_z with the x space
E_{xz}	Projection of the augmented ellipsoid E_z to the x space
E_z	Augmented invariant ellipsoid
$F(s)$	Transfer function of the closed-loop system
F_b	Damping force
F_k	Spring force
F_m	Inertial force
$G(s)$	Transfer function of the controller

$H(s)$	Transfer function of the plant
J	Cost and cost function
J_{nc}	Finite horizon cost, implying n_c optimization variables
J_{NRMPC}	Infinite horizon cost in the NRMPC formulation given as $J_{\text{NRMPC}} = \mathbf{f}_k^T\,\mathbf{f}_k$
J_T	Terminal cost $J_T = x_{n_c}^T \mathbf{P}_f x_{n_c}$
J_∞	Infinite horizon cost $J_\infty = J_{n_c} + J_T$
K	Control gains in constant gain velocity feedback
K_p	Proportional gain in a PID controller
K_i	Integral gain in a PID controller
K_d	Derivative gain in a PID controller
M	Bending moment
M_w	Moment magnitude scale
N	Number of elements in a series, final element
T	Shear force
T_c	Time constant of a first order transfer
T_d	Derivative time constant
T_i	Integral time constant
T_s	Sampling period
V	Lyapunov function

Latin Bold Lower Case Letters

$\mathbf{a}$	Active set
$\mathbf{a}^*$	Active set at the solution
$\mathbf{b_c}$	Inequality constraint vector
$\mathbf{b_e}$	Equality constraint vector
$\mathbf{c_E}$	Stiffness coefficient in a constant electric field
$\mathbf{d}$	Piezoelectric coupling matrix
$\mathbf{e}$	Vector of control errors in tracking
$\mathbf{e_i}$	i-th column of the identity matrix
$\mathbf{e_p}$	Piezoelectric coupling matrix in the stress-charge form
$\mathbf{f}$	Vector of future perturbations
$\mathbf{g}$	Magnetostrictive coupling matrix
$\mathbf{h}$	Vertex representation of regions $\mathscr{X}_i$ in MPMPC
$\mathbf{j}$	Search directions
$\mathbf{k}$	Product of $\mathbf{V_s}\Phi\mathbf{V_s}^{-1}$ in simplified polyhedral target sets
$\mathbf{l}$	Product of $\mathbf{V_s}x$ in simplified polyhedral target sets
$\mathbf{m^i}$	Vector aiding the simplification of the NRMPC online process
$\mathbf{m_v}$	Vector aiding the simplification of the NRMPC online process
$\mathbf{o}$	Definition vector of the piecewise-affine MPC law in MPMPC
$\mathbf{p}$	Electric degrees of freedom
$\tilde{\mathbf{p}}$	Definition vector of the half space bounding region $\mathscr{X}_i$ in MPMPC

$\mathbf{q}$	Vector of displacements
$\dot{\mathbf{q}}$	Vector of velocities
$\ddot{\mathbf{q}}$	Vector of accelerations
$\mathbf{r}$	Vector of reference values
$\mathbf{s}$	Vector of Laplace operators
$\mathbf{s_H}$	Mechanical compliance at constant magnetic field
$\mathbf{s_E}$	Mechanical compliance at constant electric field
$\mathbf{u}$	Sequence of inputs
$\tilde{\mathbf{u}}$	Tail of the sequence of inputs **u**
$\mathbf{u_d}$	Vector of desired inputs in tracking
$\mathbf{u_e}$	Vector of input errors in tracking
$\mathbf{u}(s)$	Vector of inputs in Laplace domain
$\Delta\mathbf{u}$	Incremental improvement in the solution of the QP
$\mathbf{u}^*$	Optimal sequence of inputs/optimal solution of a constrained optimization problem
$\mathbf{u}^\Delta$	Optimal solution of an unconstrained optimization problem
$\mathbf{v}$	Vector aiding the simplification of the NRMPC online process
$\mathbf{v}_{\mathrm{fil}}$	Vector of individual filter outputs
$\mathbf{v}(s)$	Vector of disturbances in Laplace domain
$\mathbf{w}$	Generic optimization variable
$\mathbf{w}(s)$	Vector of references in Laplace domain
$\mathbf{x}$	State vector
$\mathbf{y}$	Output vector
$\mathbf{y}(s)$	Vector of outputs in Laplace domain
$\mathbf{z}$	Augmented state vector

Latin Bold Capital Letters

$\mathbf{A}$	State transition matrix
$\mathbf{A_c}$	Inequality constraint matrix
$\mathbf{A_c^a}$	Matrix of active inequality constraints at the current active set $\mathbf{a}_p$
$\mathbf{A_e}$	Equality constraint matrix
$\mathbf{B}$	Input matrix
$\mathbf{B_0}$	Inequality constraint matrix
$\mathbf{B_d}$	Structural damping matrix
$\mathbf{B_m}$	Magnetic flux
$\mathbf{B_p}$	Dielectric loss
$\mathbf{C}$	Output matrix
$\mathbf{D}$	Direct input–output feed-through matrix
$\mathbf{D}_{\mathrm{e}}$	Electric displacement
$\mathbf{E}$	Vector selecting the first (block) element/augmented dynamics vector in NRMPC
$\mathbf{E_e}$	Electric field strength
$\mathbf{E}_{\mathrm{fil}}$	Rectangular matrix to use more filters than actuators

$\mathbf{F}$	State cost component matrix
$\mathbf{G}$	Constant in the QP problem/mixed input and state cost component matrix
$\mathbf{H}$	Hessian in the QP problem/input cost component matrix
$\mathbf{H_m}$	Magnetic field strength
$\mathbf{I}$	Matrix with ones on the main diagonal, *eye* matrix
$\mathbf{K}$	Linear quadratic gain
$\mathbf{K_p}$	Anisotropic permittivity
$\mathbf{K_s}$	Stiffness matrix
$\mathbf{K_z}$	Piezoelectric coupling matrix
$\mathbf{L}$	Cost transformation matrix
$\mathbf{L_c}$	Perturbation partition of the cost transformation matrix
$\mathbf{L_{cx}}$	Mixed perturbation-state partition of the cost transformation matrix
$\mathbf{L_e}$	Electrical input to the piezoelectric dynamics equation
$\mathbf{L_x}$	State partition of the cost transformation matrix
$\mathbf{L_{xc}}$	Mixed state-perturbation partition of the cost transformation matrix
$\mathbf{M}$	Free state prediction matrix, except in Chap. 2 where it refers to the concept of mass matrix
$\tilde{\mathbf{M}}$	Matrix aiding the mathematical transformation of the optimized augmented dynamics in NRMPC into a convex optimization problem
$\mathbf{N}$	Forced state prediction matrix
$\tilde{\mathbf{N}}$	Matrix aiding the mathematical transformation of the optimized augmented dynamics in NRMPC into a convex optimization problem
$\mathbf{O}$	Definition matrix of the piecewise-affine MPC law in MPMPC
$\mathbf{P}$	Solution of the (discrete-time algebraic) Ricatti equation
$\mathbf{P}_f$	Terminal weighting matrix
$\tilde{\mathbf{P}}$	Definition matrix of the half space bounding region $\mathscr{X}_i$ in MPMPC
$\mathbf{Q}$	State penalty weighting matrix
$\mathbf{R}$	Input penalty weighting matrix
$\mathbf{S}$	Strain
$\mathbf{S_v}$	Eigenvalues in a vector form
$\mathbf{T}$	Perturbation shift matrix in the augmented state representation
$\mathbf{U}$	Matrix aiding the mathematical transformation of the optimized augmented dynamics in NRMPC into a convex optimization problem
$\mathbf{V}$	Matrix aiding the mathematical transformation of the optimized augmented dynamics in NRMPC into a convex optimization problem
$\mathbf{W_i}$	Concatenating, truncating and simplifying matrices in the online NRMPC

$\mathbf{V}s$	Definition matrix of the simplified polyhedral target set
$\mathbf{X}$	Matrix aiding the mathematical transformation of the optimized augmented dynamics in NRMPC into a convex optimization problem
$\mathbf{Y}$	Matrix aiding the mathematical transformation of the optimized augmented dynamics in NRMPC into a convex optimization problem
$\mathbf{Z}$	Generic optimization variable

Latin Calligraphic Letters

$\mathscr{D}$	Block diagonal matrix with I and R on its main diagonal
$\mathscr{H}_\infty$	$\mathscr{H}_\infty$ optimal control gain
$\mathscr{H}_2$	$\mathscr{H}_2$ optimal control gain
$\mathscr{K}$	Unconstrained MPC gain
$\mathscr{L}$	Laplace transform
$\mathscr{Q}$	Multiple DOF displacements in Laplace domain
$\mathscr{P}$	Transfer function of the plant
$\mathscr{R}$	Block diagonal matrix with input penalty $\mathbf{R}$ on the main diagonal
$\mathscr{S}$	Sensitivity function
$\mathscr{T}$	Complementary sensitivity function
$\mathscr{X}$	Polyhedral region
$\tilde{\mathscr{X}}$	Polyhedral region after clipping
$\mathscr{X}_u$	Unsaturated polyhedral region
$\mathscr{X}_s$	Saturated polyhedral region
$\mathscr{Y}$	Convex polytope
$\mathscr{Z}$	Z-transform

Blackboard Bold Letters

$\mathbb{N}$	Set of natural numbers
$\mathbb{Z}$	Set of integers
$\mathbb{Q}$	Set of rational numbers
$\mathbb{R}$	Set of real numbers

Others

Designation and meaning of other variables, marks, indices, sub- and superscripts are defined locally. Select variables may have more than one meaning, please refer to the context for explanation. Select concepts may be marked with different variables, depending on whether it is a theoretical consideration, implementation example or actual code segment.

Relation signs $>$, $\geq$ and $<$, $\leq$ are used in the scalar binary relation sense (larger, larger or equal, smaller, smaller or equal) for scalar values and element-wise relation sense for vectors. In case the equation uses matrix notation and can be considered a linear matrix inequality, relation signs $>$, $\geq$ are used to define positive definiteness and positive semidefiniteness, while signs $<$, $\leq$ denote the concepts of negative definiteness and negative semidefiniteness. Although linear matrix inequalities in mathematics generally use the notation $\succ$, $\succeq$ and $\prec$, $\preceq$ the use of these symbols will be avoided in this book.

The notation $\kappa\{\mathbf{A}\}$ denotes the eigenvalues of matrix $\mathbf{A}$. Although most scientific literature uses the notation λ for eigenvalues, due to the interdisciplinary nature of this book the Greek letter of is λ is reserved for a different concept.

Depending on the context the number 1 may denote the mathematical concept of a vector of ones, which is an n elements long vector containing only the numbers 1: $1 = \mathbf{1} = [1\ 1\ 1 \ldots 1]^T$. This book does not utilize the bold notation $\mathbf{1}$ for a vector of ones.