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Control of Linear Systems with Regulation and Input Constraints

With 28 Figures

Springer

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This book is dedicated to:

Ann, Ingmar, and Ula

(Ali Saberi)

Suseela, Aruna, Mohan, Arun, and Pavan

(Pedda Sannuti)

Preface

Ever since the beginnings of mankind eons ago, the desire to control, regulate, and track even under persistent disturbances has been a dominating influence in the development of human civilization. It is so also in the development of *Automatic Control Theory and its Applications*. The subject of output regulation occupies a central theme in all endeavors of theoreticians and practitioners alike. Yet there is no book or monograph that brings all essential modern developments on output regulation under a single cover. This book is intended to fill this void.

Main topics that are brought together in this book include among others, classical exact output regulation of linear systems along with its different facets of well-posedness, internal model principle, and structural stability; output regulation of linear systems with input amplitude and rate saturation constraints; output regulation with transient performance specifications; performance issues (such as H_2 , H_∞ , L_1 and others) with an output regulation constraint; generalized output regulation in which the set of tracking signals as well as the set of disturbances that act on the plant are broadened beyond those that are common in classical output regulation, and thus permitting us to deal with exact as well as almost output regulation under a variety of controllers, etc.

This book is designed to meet the needs of a variety of audiences including practicing control engineers, graduate students, and researchers in control engineering. Also, it is written to be suitable for self reading and appropriate as a textbook or prime reference when teaching a first or second year graduate course in control theory. Recommended background for this book is (a) linear algebra and matrix theory, (b) linear differential equations, (c) a course in linear systems and state-space methods, and (d) a course in linear control theory.

No work of this magnitude can be undertaken without any sacrifices. Our families are the ones who sacrificed the time we could have spent with them,

thus allowing us to complete this work. We owe a debt of gratitude to our families, and it is natural that we dedicate this book to them.

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