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Modelling and Identification in Robotics

With 50 Figures

Springer

Professor Krzysztof Kozlowski
Faculty of Electrical Engineering, Technical University of Poznan,
Ul. Piotrowo 3a, 60-965 Poznan.

ISBN 978-1-4471-1139-9 ISBN 978-1-4471-0429-2 (eBook)
DOI 10.1007/978-1-4471-0429-2

British Library Cataloguing in Publication Data
Kozlowski, Krzysztof
Modelling and identification in robotics. - (Advances in
industrial control)
1. Robotics 2. Robots - Dynamics - Computer simulation
3. Robots - Programming
1. Title
628.8'92'0113
ISBN 978-1-4471-1139-9

Library of Congress Cataloging-in-Publication Data
Kozlowski, Krzysztof, 1951-
Modelling and identification in robotics / Krzysztof Kozlowski.
p. cm. -- (Advances in industrial control)
Includes bibliographical references (p.).
ISBN 978-1-4471-1139-9
1. Robotics--Mathematical models. 2. Robots--Dynamics-
-Mathematical models. 3. System identification. I. Title.
II. Series.
TJ211.K69 1998 97-51769
629.8'92--dc21 CIP

Originally published by Springer-Verlag London Limited 1998

Softcover reprint of the hardcover 1st edition 1998

Typesetting: Camera ready by authors
69/3830-543210 Printed on acid-free paper

Advances in Industrial Control

Professor Dr -Ing M. Thoma
Westermannweg 7
D-30419 Hannover
Germany

Professor H. Kimura
Department of Mathematical Engineering and Information Physics
Faculty of Engineering
The University of Tokyo
7-3-1 Hongo
Bunkyo Ku
Tokyo 113
Japan

Professor A.J. Laub
Department of Electrical and Computer Engineering
University of California
Santa Barbara
California 93106
United States of America

Professor J.B. Moore
Department of Systems Engineering
The Australian National University
Research School of Physical Sciences
GPO Box 4
Canberra
ACT 2601
Australia

Dr M.K. Masten
Texas Instruments
2309 Northcrest
Plano
TX 75075
United States of America

Professor Ton Backx
AspenTech Europe B.V.
De Waal 32
NL-5684 PH Best
The Netherlands

SERIES EDITORS' FOREWORD

The series *Advances in Industrial Control* aims to report and encourage technology transfer in control engineering. The rapid development of control technology impacts all areas of the control discipline. New theory, new controllers, actuators, sensors, new industrial processes, computing methods, new applications, new philosophies…, new challenges. Much of this development work resides in industrial reports, feasibility study papers and the reports of advanced collaborative projects. The series offers an opportunity for researchers to present an extended exposition of such new work in all aspects of industrial control for wider and rapid dissemination.

The robotics control field readily exemplifies the multi-disciplinary nature of control engineering. In this monograph by Krzysztof Kozlowski are the science of mechanics, and computer science and the engineering disciplines of control, electronics and mechanical engineering. The work has an objective of deriving and validating robotic arm models for different types of industrial robots. Consequently the methods of model identification play a significant role in the research reported. How to do practical parameter identification is always a mixture of theory, computational consideration, insight and art. The way in which this combination has come together to yield consistent and repeatable results is well demonstrated in this monograph. The work concluded with a chapter on approaches to position control for different robot types.

The systematic approach to robots engineering, model derivation, parameter estimation and robot control should be useful for the student and the expert engineer alike. However, the practical insights reported should be especially valuable to all engineers working with robotic systems.

M.J. Grimble and M.A. Johnson
Industrial Control Centre
University of Strathclyde,
Glasgow, G1 1QY.
Scotland, UK.

Krzysztof Kozłowski

Modelling and Identification in Robotics

September 2, 2008

Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo
Hong Kong Barcelona
Budapest

To my wife,
Wanda,
and children,
Stanisław and Marta

Preface

The idea of this monograph was conceived when I wrote my Habilitation Thesis entitled ”Mathematical dynamic robot models and identification of their parameters” [122] concerning theoretical aspects of robot dynamics modelling and identification of their parameters. I have been interested in robotics for the last ten years. My first project in robotics entitled ”Optimisation methods for nonlinear robot control”, was for the Institute of Fundamental Problems in Technology, Polish Academy of Sciences. The results of this work are presented in four reports [97, 98, 112, 119]. In these reports the theoretical aspects of robot modelling and identification of dynamic parameters of robot models are presented. Results of this work were published later in the Habilitation Thesis [122]. In the thesis dynamic equations of motion for an open kinematic chain consisting of n rigid bodies are formulated. The equations of motion are formulated based on the Lagrangian and Newton-Euler formalism. These equations are presented in a form which is suitable for the purpose of identification of their parameters. The parameters which appear in these equations are: mass, centre of mass (moment of the first order) and elements of the inertia tensor of each individual rigid body. As a result of this formulation, a dynamic model of a robot, which consists of dynamic parameters which are linear combinations of the link dynamic parameters, is obtained. These models are know as canonical models which are expressed in terms of a minimum number of dynamic parameters. In the Habilitation Thesis, possible ways to obtain the canonical models are described. In addition, the theoretical results presented in [122] are illustrated by simulation considerations which were satisfactory, but only from a theoretical point of view.

Robotics is an interdisciplinary field and involves such disciplines as control theory, mechanics, computer science and electronics. My background is control engineering and I have been working in this practical field for many years. I was interested in obtaining experimental results which would verify robotic theories concerning the dynamics of robots and the identification of their parameters. I started to build an experimental set–up around the industrial IRp-6 robot in our Robot Control Laboratory at the Chair of Control, Robotics, and Computer Science, Poznan University of Technology, Poland. The IRp-6 robot at that time was available on the Polish market and we decided to buy it both for teaching and research purposes. Unfortunately, this robot has a closed architecture and the manufacturer, Factory for Automation, Ostrów Wielkopolski, Poland did not want to provide us with any detailed technical information about the robot. The Institute of Control Engineering, Technical University of Warsaw built a bus access interface [144] between the robot controller and the host computer and also a data acquisition system which collects motor positions, velocities, and currents of the IRp-6 robot [143]. Our experimental set–up is equipped with these two devices. In addition we bought a force/torque sensor JR3 [93] which allows us to collect forces and torques for the purpose of load identification and hybrid control.

A supporting programming system which performs various functions was written for the experimental set–up. Several experiments were carried out, most of them by Dr. P. Dutkiewicz while writing his Ph.D. Thesis, under my supervision [50], entitled "Robot and load identification of the IRp-6 robot with hybrid control and programming elements". Parts of my Habilitation Thesis and Dr. P.Dutkiewicz's Thesis are in the book entitled "Modelling and identification in robotics" published in Polish by the Poznan University of Technology Press [139]. In both references many experimental results concerning the practical identification of robot and load dynamics and hybrid control are included. A lot of attention was put on optimal trajectory design for the purpose of identification in robotics and planning the experiments. These problems are very interesting in the robotics field since, in general, the identification of dynamic parameters of the mechanical systems is not a trivial problem. The last two references are a logical consequence of the theoretical considerations presented in my Habilitation Thesis. I strongly believe that the experimental results constitute a good illustrations of the theory. It is hard to consider theoretical results in such a practical and interdisciplinary field as robotics. It was my main motivation in writing this monograph, while at the same time I believe that I have maintained a reasonable balance between theory and practice throughout.

Most of the robots in industrial practice are geared robots. Therefore the results which are included in the monograph are devoted to this type of robot. Identification of their dynamic parameters is not a trivial problem due to friction phenomena. I believe that friction torques versus velocity have to be precompensated and these torques cannot be neglected. Friction depends on many factors which are sometimes difficult to describe analytically. This problem is described in the monograph. Due to the fact that geared robots are equipped with gears, the dynamic effects are to some extent decoupled because the inertia of the rigid body, seen from the motor side is divided by the square of the gear ratio. At the same time, friction phenomena within the transmission mechanisms cannot be neglected. In general, geared robots do not move very fast and this further complicates the practical identification of their dynamic parameters

Another type of robot used in industry is the direct drive robot (DDA) which is not as popular as the geared robot. These robots are usually faster than geared robots and their dynamics cannot be neglected. Although the friction effects are not as dominant as for geared robots they cannot be neglected. As an example of a robot of the first type we took the IRp-6 robot and of the second type EDDA (Experimental Direct Drive Robot) [124] built for research purposes. These two robots were used to carry out experimental work concerning robot and load dynamics identification. The experimental results were compared to those existing in robotics literature. In order to carry out the experiments on an open architecture, an experimental set–up of a one link geared robot was constructed in the Robot Control Laboratory. The identification experiments performed on it and this set-up was designed for teaching and research purposes. These results are also compared with the simulation results concerning these robots. I believe that the comparison is wide and applies to recently obtained results in the field of robotics.

The experimental work was made possible due to the kind support of the State Committee for Scientific Research (SCSR). Two research projects strictly connected to the contents of the monograph were carried out under my supervision. The first project entitled "Programming system of the IRp-6 ASEA robot, its model and force and torque sensor feedback", No 1096/S5/92/06 [34] was carried out in 1992. The second project supported by the SCSR was entitled "Acquisition, knowledge processing, and parallel computations of the optimal trajectory for robots", No 8S505 009.06 [133] was carried out in 1994-95. Besides that, a small part of the research work was supported by the Poznan University of Technology under the project "Implementation of a force and torque sensor in hybrid controller of the IRp-6 industrial robot" [127] in 1994. Under these three projects it was possible to buy specialised and expensive equipment which was necessary to perform the experiments described in the monograph. Further support came from the for Foundation for Promotion and Advanced Automation Technology, a Japanese organisation which sponsored the project entitled "Development and experimental validation of real–time inverse and forward dynamics algorithms for an industrial robot" [126] and also assisted with equipment and travelling expenses for me to attend the IEEE Robotics and Automation Conference, of one the most important conferences in the field of robotics. These scientific contacts played a very important role in the research carried out at our laboratory.

I would like to state clearly that without the support of the above institutions and Japanese organisation the experimental part of the research would be not possible. I should like to express my gratitude and thanks for their support.

The experimental work concerning the EDDA robot was performed in the Institut für Robotik und Prozessiformatik, Technische Universität Braunschweig, Germany. I was there under the DAAD (Deutscher Akademischer

Austauschdienst) in TU Braunschweig from 1st October, 1991 to 28th February, 1992. The experimental set–up was built by the German side and the experiments were carried out together with Dr. M. Prüfer. I should like to express my gratitude and thanks to, the DADD organisation and to Dr. M. Prüfer for his joint research effort.

Finally, it gives me genuine pleasure to acknowledge some people who have made this monograph possible. I express my thanks to my colleague Dr. P. Dutkiewicz for carrying out most of the experimental work presented in the monograph. His comments on the draft version of the monograph were very valuable. I should also like to thank many of my graduate students who wrote parts of the programming system and took part in experimental work too. Now they work in different fields in industry and factory automation.

I am very grateful to Mr. K. Romanowski, M.S. for his suggestions concerning the merits of the monograph and improving my English. I am also grateful to Mr. G. Niwczyk, M.S. for his help and patience and the typeset of the monograph.

Prof. M. J. Grimble and Dr. M. J. Johnson editors of the Advances in Industrial Control Monograph Series are gratefully acknowledged for their encouragement in pursuing this project.

Last but not least, thanks are due to Mr. G. Cash for reading the monograph and polishing the English that had initially been "Polished" by me in the early version of the monograph.

Poznań, December, 1997 Krzysztof R. Kozłowski

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Nomenclature

$a_i, d_i, \alpha_i, \theta_i$ – parameters of the modified Denavit-Hartenberg notation,

$\boldsymbol{A}(\boldsymbol{q})$ – moment of inertia matrix with dimensions $n \times n$, symmetric and nonsingular, with elements A_{ij},

$\boldsymbol{B}(\boldsymbol{q})$ – $n \times (n\,(n-1)\,/2)$ Coriolis coefficient matrix with elements $B_{ijk} = \frac{\partial A_{ij}}{\partial q_k} + \frac{\partial A_{ik}}{\partial q_j} - \frac{\partial A_{jk}}{\partial q_i}$,

$\bar{\boldsymbol{C}}(\boldsymbol{q})$ – $n \times n$ centrifugal coefficient matrix with elements $\bar{C}_{ij} = \frac{\partial A_{ij}}{\partial q_j} - \frac{1}{2}\frac{\partial A_{jj}}{\partial q_i}$,

$\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ – $n \times n$ Coriolis matrix with elements C_{ij},

${}^{i}\boldsymbol{c}_i$ – vector connecting the origin of the coordinate frame i with the centre of mass of rigid body i, left superscript denotes that this vector is expressed in the coordinate frame i, therefore when the manipulator moves the three components of this vector are constant,

c_i – stiffness constant of the drive system i,

C_i – centre of mass of the rigid body i,

dd_{ir} – differential of the function d_{ir},

$\boldsymbol{d}^T(\boldsymbol{q}, \dot{\boldsymbol{q}})$ – vector function with dimensions $1 \times 12n$ which appears in the integral model, for the canonical integral model its dimensions are $1 \times m_c$; this function is also named as the regression function for of integral model,

$\boldsymbol{D}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$ – matrix function with dimensions $n \times 10n$ which appears in the differential model, it is also called as the input regression function of the differential model; for the differential canonical model this matrix has dimensions $n \times m_c$,

$\boldsymbol{dl}^T(\boldsymbol{q}, \dot{\boldsymbol{q}})$ – vector function with dimensions $1 \times 10n$ which appears in the integral model without viscous and Coulomb friction coefficients,

$\boldsymbol{df_s}^T = \boldsymbol{df_s}^T = [\int\limits_{t_1}^{t_2} |\dot{q}_1| dt,\ \cdots\ , \int\limits_{t_1}^{t_2} |\dot{q}_n| dt]$ – vector of the function associated with vector $\boldsymbol{f}_s$ in the integral model,

$\boldsymbol{df_v}^T = \boldsymbol{df_v}^T = [\int\limits_{t_1}^{t_2} \dot{q}_1^2 dt,\ \cdots\ , \int\limits_{t_1}^{t_2} \dot{q}_n^2 dt]$ – vector of the function associated with vector $\boldsymbol{f}_v$ in the integral model,

DDA – Direct Drive Arm,

ΔT – time increment,

E – expectation operator,

E_{ki} – kinetic energy rigid body i of the manipulator,

E_{kc} – total kinetic energy of the manipulator,

$E_{kc}(t_i)$ – total kinetic energy of the manipulator at time instant t_i

E_{pi} – potential energy of rigid body i of the manipulator,

E_{pc} – total potential energy of the manipulator,

$E_{pc}(t_i)$ – total potential energy at time instant t_i,

$\boldsymbol{C}_v$ – $diag[\dot{q}_1, \dot{q}_2 \dots \dot{q}_n]$ diagonal $n \times n$ matrix of the functions associated with the vector of velocity friction,

$\boldsymbol{C}_s$ – $diag[sgn(\dot{q}_1), sgn(\dot{q}_2) \dots sgn(\dot{q_n})]$ diagonal $n \times n$ matrix of the functions associated with the Coulomb friction coefficients,

$f(k\Delta T) = f[\boldsymbol{s}(k\Delta T), \ddot{\boldsymbol{q}}(k\Delta T)]$ – function which describes the dynamical behaviour of the system for the optimisation procedure of the input trajectory for the differential model. For the integral model, this function is a function of generalised positions and velocities,

F_{id} – coefficient which is a difference between the break away friction coefficient and the Coulomb friction coefficient for joint i,

F_{is} – viscous friction coefficient for joint i,

F_{iv} – velocity friction coefficient (or viscous friction coefficient) for joint i,

$\boldsymbol{f}_s$ – $n \times 1$ vector of the Coulomb friction coefficients for all manipulator joints,

$\boldsymbol{f}_v$ – $n \times 1$ vector of the velocity friction coefficients for all manipulator joints,

$\frac{\partial F_2}{\partial \phi_2}, \frac{\partial F_3}{\partial \phi_3}$ – nonlinear functions which describe gear ratios between joint and motor velocities of the second and third links of the IRp-6 robot. These functions depend on the geometry of the parallel structure of the IRp-6 robot and joint displacement of the second and third link, respectively,

$\boldsymbol{f}$ – spatial force exerted by a force/torque sensor on the body in point P,

FFT – Fast Fourier Transform,

$\boldsymbol{\Phi}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$ – input regression matrix in the canonical differential model written as the upper triangular matrix with elements $\boldsymbol{\Phi}_{ij}^T$,

$\phi_1, \phi_2, \phi_3, \phi_4, \phi_5$ – generalised joint coordinates of the IRp-6 robot (see Fig 3.6b) and Table 3.2,

$\dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3, \dot{\phi}_4, \dot{\phi}_5$ – generalised joint velocities of the IRp-6 robot,

G_i – component i of the vector of the gravitational forces,

$\boldsymbol{\Gamma}_i$ – an update gain matrix (also called the adaptation gain matrix).

$\boldsymbol{g} = [0, 0, g_0]^T$ – gravity acceleration vector, where $g_0 = -9,81$ m/s^2,

$\boldsymbol{G}(\boldsymbol{q})$ – $n \times 1$ vector of gravitational forces,

$H(t_i) = E_{kc}(t_i) + E_{pc}(t_i)$ – sum of the total kinetic and potential energy of the manipulator at time instant t_i,

$H(k\Delta T)$ – Hamiltonian at instant time k,

${}^i\boldsymbol{I}_i$ – inertia tensor of link i (or second order moment) expressed in local coordinate frame i, therefore it is constant when the manipulator moves,

$\boldsymbol{I}_c$ – inertia tensor of the load with respect to the centre of mass of the load expressed in the coordinate frame with origin at the centre of mass,

I_{ia} – inertia moment of the actuator i,

$\boldsymbol{J}$ – Jacobian of a manipulator,

${}^{i}\boldsymbol{J}_i$ – pseudo inertia matrix of link i expressed in the local coordinate frame, and therefore is constant when the manipulator moves,

$\boldsymbol{K}_p$ – global compliance matrix of the manipulator,

$\boldsymbol{K}_p^{-1}$ – global stiffness matrix of the manipulator,

${}^{i}\boldsymbol{l}_{i+1}$ – vector which connects the origin of coordinate frame i and $(i+1)$ expressed in coordinate frame i,

$L(\cdot)$ – the Laplace transformation of a function $(\cdot)$,

$L^{-1}(\cdot)$ – inverse of the Laplace transformation,

$\boldsymbol{\lambda}$ – vector of Lagrange's multipliers,

$\mathcal{L}$ – Lagriangian,

m_i – mass of rigid body i,

$m_i {}^{i}\boldsymbol{c}_i = [m_i c_{ix}, m_i c_{iy}, m_i c_{iz}]^T$ – first order moment of rigid body i expressed in the local coordinate frame i, note that components of this vector are calculated as a result of the multiplication of the mass m_i by the vector of centre mass of body i,

m_c – number of parameters in the canonical model,

n – number of degrees of freedom of the manipulator,

$\boldsymbol{m}$ – torque vector exerted by the force/torque sensor on the rigid body,

$\boldsymbol{n}, \boldsymbol{o}, \boldsymbol{a}$ – three orthonormal vectors associated with the tool centre point, $\boldsymbol{n}$ - normal vector, $\boldsymbol{o}$ - orientation vector, and $\boldsymbol{a}$ - approach vector,

O_i – origin of coordinate frame i,

$\boldsymbol{P}$ – input correlation matrix,

$\Psi_1, \Psi_2, \Psi_3, \Psi_4, \Psi_5$ – motor joint variables for the IRp-6 robot,

$\dot{\Psi}_1, \dot{\Psi}_2, \dot{\Psi}_3, \dot{\Psi}_4, \dot{\Psi}_5$ – motor joint velocities of the IRp-6 robot,

q_i – joint i generalised position,

$\dot{q}_i$ – joint i generalised velocity,

$\ddot{q}_i$ – joint i generalised acceleration,

$\boldsymbol{q}$ – $n \times 1$ vector of generalised positions,

$\dot{\boldsymbol{q}}$ – $n \times 1$ vector of generalised velocities,

$\ddot{\boldsymbol{q}}$ – $n \times 1$ vector of generalised accelerations,

$\boldsymbol{Q}_i(q_i)$ – friction force acting at joint i,

r – current number of the basis function,

${}^{i-1}_{i}\boldsymbol{R}$ – direction cosine matrix between coordinate frames i and $(i-1)$,

$\boldsymbol{s} = [\boldsymbol{q}, \dot{\boldsymbol{q}}]$ – state of the system for the differential model; in the case of the integral model $\boldsymbol{s} = \boldsymbol{q}$,

$sgn(\cdot)$ – sign function of the argument $(\cdot)$,

$Tr(\cdot)$ – trace of the matrix $(\cdot)$,

$\sigma_i = 0$ – denotes that the joint i is rotational,

$\sigma_i = 1$ – denotes that the joint i is translational,

$\bar{\sigma}_i = 1 - \sigma_i$,

$(\cdot)^T$ – transpose operation,

τ_i – joint i generalised force,

$\boldsymbol{\tau}$ – $n \times 1$ vector of generalised forces,

$\boldsymbol{\tau}_e$ – $n \times 1$ vector of non-potential forces (or equivalently non-dissipative forces at the system),

τ_{ifs} – Coulomb friction force or torque at joint i,

$\boldsymbol{\tau}_{fs}$ – $n \times 1$ vector of the Coulomb friction forces or torques for all the joints of the manipulator,

τ_{ifv} – viscous friction force or torque at joint i,

$\boldsymbol{\tau}_{fv}$ – $n \times 1$ vector of viscous forces or torques for all the joints of the manipulator,

${}^i\boldsymbol{v}_i = [v_{ix}, v_{iy}, v_{iz}]^T$ – 3×1 vector of linear velocity of the origin of the coordinate frame i,

$\boldsymbol{w}_s$ – 6×1 vector of 3 forces and 3 torques; (or equivalently wrench),

$\boldsymbol{x}$ – $12n \times 1$ vector consisting of dynamic parameters of all links and Coulomb and velocity friction coefficients for all joints of the manipulator,

$\boldsymbol{x}'$ – $10n \times 1$ vector of dynamic parameters of the manipulator,

$\boldsymbol{x}'_i$ – 10×1 vector consisting of 10 individual link i dynamic parameters namely; mass, three components of the first order moment and six components of the inertia tensor,

$\boldsymbol{x}'_{ic}$ – vector consisting of agregated parameters of link i which has number less or equal to 10,

$\boldsymbol{x}'_c$ – $m_c \times 1$ vector of the aggregated parameters of the canonical model (vector with a minimum number of dynamic parameters),

$\hat{\boldsymbol{x}}$ – 12×1 vector of the estimates of the parameter vector $\boldsymbol{x}$,

$\boldsymbol{X}_i$ – dynamic parameter i which appears in the vector of dynamic parameters,

$\hat{\boldsymbol{X}}_i$ – estimate of parameter $\boldsymbol{X}_i$,

${}^i\boldsymbol{\omega}_i = [\omega_{ix}, \omega_{iy}, \omega_{iz}]^T$ – angular velocity of link i,

$y = \int \boldsymbol{\tau}^T \dot{\boldsymbol{q}} dt$ – integral of the mechanical energy,

$y = F(\boldsymbol{P}_K)$ – cost function in the optimisation procedure, F can be either the condition number or the reciprocal of the smallest singular value or the determinant of the input correlation matrix,

$\boldsymbol{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$ – input regression matrix in the differential model written as the upper triangular matrix with elements $\boldsymbol{Y}_{ij}^T$,