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Information Algebras

Generic Structures For Inference

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Preface

What is the book about?

Information is a central concept of science, especially of Computer Science. The well developed fields of statistical and algorithmic information theory focus on measuring information content of messages, statistical data or computational objects. There are however many more facets to information. We mention only three:

1. Information should always be considered relative to a given specific question. And if necessary, information must be focused onto this question. The part of information relevant to the question must be extracted.

2. Information may arise from different sources. There is therefore the need for aggregation or combination of different pieces of information. This is to ensure that the whole information available is taken into account.

3. Information can be uncertain, because its source is unreliable or because it may be distorted. Therefore it must be considered as defeasible and it must be weighed according to its reliability.

The first two aspects induce a natural algebraic structure of information. The two basic operations of this algebra are combination or aggregation of information and focusing of information. Especially the operation of focusing relates information to a structure of interdependent questions. This book is devoted to an introduction into this algebraic structure and its variants. It will demonstrate that many well known, but very different formalisms such as for example relational algebra and probabilistic networks, as well as many others, fit into this abstract framework. The algebra allows for a generic study of the structure of information. Furthermore it permits to construct generic architectures for inference.

The third item above, points to a very central property of real-life information, namely uncertainty. Clearly, probability is the appropriate tool to

describe and study uncertainty. Information however is not only numerical. Therefore, the usual concept of random variables is not sufficient to study uncertain information. It turns out that the algebra of information is the natural framework to model uncertain information. This will be another important subject of this book.

Surely the algebra of information has links to statistical and algorithmic information theory. This very important synthesis is yet to be worked out. It promises to be a fruitful area of research.

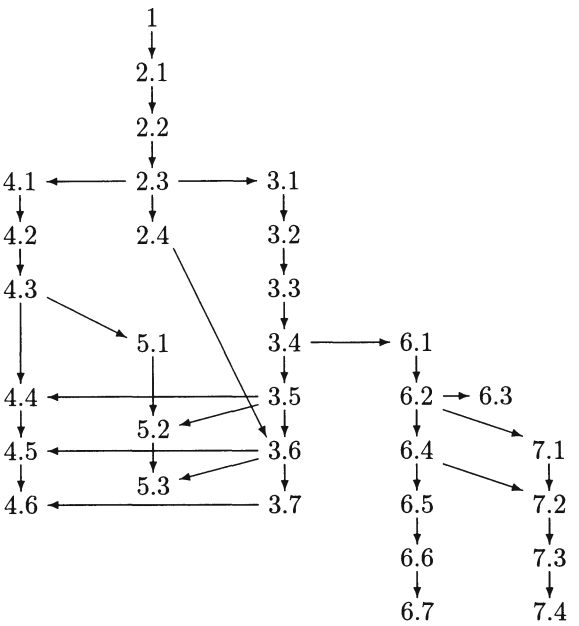
This book has its root in the past European Basic Research activity no. 6156 DRUMS 2 (Defeasible Reasoning and Uncertainty Management). In this project, scientists from different fields like philosophy, formal logic, probability theory, statistics, evidence theory and fuzzy systems met to discuss approaches to uncertainty. It was in this stimulating environment that the desire for a unifying treatment of apparently quite different formalisms arose. The foundation for it was fortunately given in a basic axiomatic system proposed in (Shenoy & Shafer 1990). This system in turn was derived from a fundamental paper on probabilities on graphical structures (Lauritzen & Spiegelhalter, 1988). For more information about this background we refer to Chapter 1.

Organization

The dependencies of the sections are shown in the chart on the next page. The material covered is organized into seven chapters:

- Chap. 1 gives a more detailed overview of the subject of this book and its background.
- Chap. 2 introduces the basic algebraic structure and presents several important examples.
- Chap. 3 develops the algebraic theory of information.
- Chap. 4 presents generic architectures for inference.
- Chap. 5 discusses the notion of conditional independence in the abstract frame of information algebra.
- Chap. 6 is devoted to a discussion of idempotent information algebras and information systems.
- Chap. 7 treats a general model of uncertain information.

As the dependency graph shows, the book need not be read in a sequential manner. Chap. 2 is basic and needed for all other parts of the book. Readers interested in computational questions may continue with Chapter 4. However, for the more advanced architectures, Chapter 3 is needed. Chapter 3 and 4 are a prerequisite for conditional independence, Chapter 5. Readers interested more in the algebraic theory can concentrate on Chapters 3 and 6. Finally the discussion of uncertain information in Chapter 7 is based on Chapter 6.



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I would also like to thank my collaborators who participated in the research leading to this book. At the first place my thanks are due to Robert Staerk, who made essential contributions to Chapter 6, which were not published. Bernhard Anrig, Dritan Berzati, Rolf Haenni, Norbert Lehmann and Paul-André Monney all made important contributions to probabilistic argumentation systems, which were our test field for the generic inference architecture

and which laid also the foundation for Chapter 7 on uncertain information. I thank Bernhard Anrig, Dritan Berzati, Norbert Lehmann and Patrick Hertelendy also for proof reading and in particular Bernhard Anrig for extremely valuable help in typesetting of this book. I thank also Springer-Verlag and in particular Stephen Bailey for their editing support.

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