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Undergraduate Texts in Mathematics

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(continued after index)

Johannes Buchmann

Introduction to Cryptography

Second Edition

Springer

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For Almut, Daniel, and Jan

Contents

Preface for the Second Edition	xiii
Preface	xv
1 Integers	1
1.1 Basics	1
1.2 Divisibility	3
1.3 Representation of Integers	4
1.4 O - and Ω -Notation	6
1.5 Cost of Addition, Multiplication, and Division with Remainder	7
1.6 Polynomial Time	9
1.7 Greatest Common Divisor	9
1.8 Euclidean Algorithm	12
1.9 Extended Euclidean Algorithm	16
1.10 Analysis of the Extended Euclidean Algorithm . . .	18
1.11 Factoring into Primes	22
1.12 Exercises	24
2 Congruences and Residue Class Rings	29
2.1 Congruences	29

2.2	Semigroups	32
2.3	Groups	34
2.4	Residue Class Ring	35
2.5	Fields	36
2.6	Division in the Residue Class Ring	36
2.7	Analysis of the Operations in the Residue Class Ring	38
2.8	Multiplicative Group of Residues mod m	39
2.9	Order of Group Elements	41
2.10	Subgroups	42
2.11	Fermat's Little Theorem	44
2.12	Fast Exponentiation	45
2.13	Fast Evaluation of Power Products	48
2.14	Computation of Element Orders	49
2.15	The Chinese Remainder Theorem	51
2.16	Decomposition of the Residue Class Ring	53
2.17	A Formula for the Euler φ -Function	55
2.18	Polynomials	56
2.19	Polynomials over Fields	58
2.20	Construction of Finite Fields	61
2.21	The Structure of the Unit Group of Finite Fields	65
2.22	Structure of the Multiplicative Group of Residues Modulo a Prime Number	66
2.23	Exercises	67
3	Encryption	71
3.1	Encryption Schemes	71
3.2	Symmetric and Asymmetric Cryptosystems	73
3.3	Cryptanalysis	74
3.4	Alphabets and Words	77
3.5	Permutations	80
3.6	Block Ciphers	81
3.7	Multiple Encryption	82
3.8	The Use of Block Ciphers	83
3.9	Stream Ciphers	93
3.10	The Affine Cipher	95
3.11	Matrices and Linear Maps	97
3.12	Affine Linear Block Ciphers	102
3.13	Vigenère, Hill, and Permutation Ciphers	103

3.14	Cryptanalysis of Affine Linear Block Ciphers	104
3.15	Secure Cryptosystems	105
3.16	Exercises	111
4	Probability and Perfect Secrecy	115
4.1	Probability	115
4.2	Conditional Probability	117
4.3	Birthday Paradox	118
4.4	Perfect Secrecy	119
4.5	Vernam One-Time Pad	123
4.6	Random Numbers	124
4.7	Pseudorandom Numbers	124
4.8	Exercises	125
5	DES	127
5.1	Feistel Ciphers	127
5.2	DES Algorithm	128
5.3	An Example	134
5.4	Security of DES	136
5.5	Exercises	137
6	AES	139
6.1	Notation	139
6.2	Cipher	140
6.3	KeyExpansion	145
6.4	An Example	146
6.5	InvCipher	148
6.6	Exercises	148
7	Prime Number Generation	151
7.1	Trial Division	151
7.2	Fermat Test	153
7.3	Carmichael Numbers	154
7.4	Miller-Rabin Test	156
7.5	Random Primes	159
7.6	Exercises	160

8	Public-Key Encryption	163
8.1	Idea	163
8.2	Security	165
8.3	RSA Cryptosystem	167
8.4	Rabin Encryption	181
8.5	Diffie-Hellman Key Exchange	186
8.6	ElGamal Encryption	191
8.7	Exercises	196
9	Factoring	199
9.1	Trial Division	199
9.2	$p - 1$ Method	200
9.3	Quadratic sieve	201
9.4	Analysis of the Quadratic Sieve	206
9.5	Efficiency of Other Factoring Algorithms	210
9.6	Exercises	211
10	Discrete Logarithms	213
10.1	The DL Problem	213
10.2	Enumeration	214
10.3	Shanks Baby-Step Giant-Step Algorithm	214
10.4	The Pollard ρ -Algorithm	217
10.5	The Pohlig-Hellman Algorithm	221
10.6	Index Calculus	226
10.7	Other Algorithms	230
10.8	Generalization of the Index Calculus Algorithm	231
10.9	Exercises	232
11	Cryptographic Hash Functions	235
11.1	Hash Functions and Compression Functions	235
11.2	Birthday Attack	238
11.3	Compression Functions from Encryption Functions	239
11.4	Hash Functions from Compression Functions	239
11.5	SHA-1	242
11.6	Other Hash Functions	244
11.7	An Arithmetic Compression Function	245
11.8	Message Authentication Codes	247
11.9	Exercises	248

12 Digital Signatures	249
12.1 Idea	249
12.2 Security	250
12.3 RSA Signatures	251
12.4 Signatures from Public-Key Systems	257
12.5 ElGamal Signature	257
12.6 The Digital Signature Algorithm (DSA)	263
12.7 Undeniable Signatures	266
12.8 Blind Signatures	271
12.9 Exercises	274
 13 Other Systems	 277
13.1 Finite Fields	278
13.2 Elliptic Curves	278
13.3 Quadratic Forms	282
13.4 Exercises	283
 14 Identification	 285
14.1 Passwords	286
14.2 One-Time Passwords	287
14.3 Challenge-Response Identification	287
14.4 Exercises	292
 15 Secret Sharing	 293
15.1 The Principle	293
15.2 The Shamir Secret Sharing Protocol	294
15.3 Exercises	297
 16 Public-Key Infrastructures	 299
16.1 Personal Security Environments	299
16.2 Certification Authorities	301
16.3 Certificate Chains	306
 Solutions of the exercises	 307
 References	 325
 Index	 331

Preface for the Second Edition

The second edition of my introduction to cryptography contains updates and new material. I have updated the discussion of the security of encryption and signature schemes and the state of the art in factoring and computing discrete logarithms. I have added descriptions of time-memory trade of attacks and algebraic attacks on block ciphers, the Advanced Encryption Standard (AES), the Secure Hash Algorithm (SHA-1), secret sharing schemes, and undeniable and blind signatures. I have also corrected the errors that have been reported to me. I thank the readers of the first edition for all comments and suggestions.

October 2003

Johannes Buchmann

Preface

Cryptography is a key technology in electronic security systems. Modern cryptographic techniques have many uses, such as to digitally sign documents, for access control, to implement electronic money, and for copyright protection. Because of these important uses it is necessary that users be able to estimate the efficiency and security of cryptographic techniques. It is not sufficient for them to know only how the techniques work.

This book is written for readers who want to learn about modern cryptographic algorithms and their mathematical foundation but who do not have the necessary mathematical background. It is my goal to explain the basic techniques of modern cryptography, including the necessary mathematical results from linear algebra, algebra, number theory, and probability theory. I only assume basic mathematical knowledge.

The book is based on courses in cryptography that I have been teaching at the Technical University Darmstadt, since 1996. I thank all students who attended the courses and who read the manuscript carefully for their interest and support. In particular, I would like to thank Harald Baier, Gabi Barking, Manuel Breuning, Safuat Hamdy, Birgit Henhapl, Michael Jacobson (who also corrected my English), Markus Maurer, Andreas Meyer, Stefan Neis, Sachar Paulus, Thomas

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Johannes Buchmann