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Edwin Hewitt Kenneth A. Ross

Abstract Harmonic Analysis

Volume I

Structure of Topological Groups

Integration Theory Group Representations

Second Edition



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Preface to the Second Edition

It has not been possible to rewrite the entire book for this Second Edition. It would have been gratifying to resurvey the theory of topological groups in the light of progress made in the period 1962—1978, to amplify some sections and curtail others, and in general to profit from our experience since the book was published. Market conditions and other commitments incurred by the authors have dictated otherwise. We have nonetheless taken advantage of the kindness of Springer-Verlag to make a number of improvements in the text and of course to correct misprints and mathematical blunders.

We are in debt to the readers who have written to us or spoken with us about the text, and we have tried to follow their suggestions. We are happy here to record our gratitude to ROBERT B. BURCKEL, W. WISTAR COMFORT, ROBERT E. EDWARDS, ROBERT E. JAMISON, JORGE M. LÓPEZ, THEODORE W. PALMER, WILLARD A. PARKER, KARL R. STROMBERG, and FRED THOELE, as well as to a host of others who have kindly made suggestions to us.

Our thanks are due as well to Springer-Verlag for their support of our work.

Seattle, Washington
Eugene, Oregon

EDWIN HEWITT
KENNETH A. ROSS

January 1979

Preface to the First Edition

When we accepted the kind invitation of Prof. Dr. F. K. SCHMIDT to write a monograph on abstract harmonic analysis for the *Grundlehren der Mathematischen Wissenschaften* series, we intended to write all that we could find out about the subject in a text of about 600 printed pages. We intended that our book should be accessible to beginners, and we hoped to make it useful to specialists as well. These aims proved to be mutually inconsistent. Hence the present volume comprises only half of the projected work. It gives all of the structure of topological groups needed for harmonic analysis as it is known to us; it treats integration on locally compact groups in detail; it contains an introduction to the theory of group representations. In the second volume we will treat harmonic analysis on compact groups and locally compact Abelian groups, in considerable detail.

The book is based on courses given by E. HEWITT at the University of Washington and the University of Uppsala, although naturally the material of these courses has been enormously expanded to meet the needs of a formal monograph. Like the other treatments of harmonic analysis that have appeared since 1940, the book is a lineal descendant of A. WEIL's fundamental treatise (WEIL [4])¹. The debt of all workers in the field to WEIL's work is well known and enormous. We have also borrowed freely from LOOMIS's treatment of the subject (LOOMIS [2]), from NAÏMARK [1], and most especially from PONTRYAGIN [7]. In our exposition of the structure of locally compact Abelian groups and of the PONTRYAGIN-VAN KAMPEN duality theorem, we have been strongly influenced by PONTRYAGIN's treatment. We hope to have justified the writing of yet another treatise on abstract harmonic analysis by taking up recent work, by writing out the details of every important construction and theorem, and by including a large number of concrete examples and facts not available in other textbooks.

The book is intended to be readable by students who have had basic graduate courses in real analysis, set-theoretic topology, and algebra as given in United States universities at the present day. That is, we suppose that the reader knows elementary set theory, set-theoretic topology, measure theory, and algebra. Our ground rule is [although this is

¹ Here and throughout the book, numbers in square brackets designate works in the bibliography found at the end of the volume. These are arranged and numbered by authors.

not hard and fast] that facts and concepts from KELLEY [2], HALMOS [2], and VAN DER WAERDEN [1] may be used without explanation or proof.

In our effort to make the book useful for specialists, we have included a large corpus of material which a beginning reader may find it wise to omit. The following suggestions are for the beginning student who wishes to get to the root of the matter as quickly as possible. First read §§ 1–7, Subheads (8.1)–(8.7), (9.1)–(9.14), and all of § 10. The reader who is already familiar with or wishes to take on faith the theory of integration on locally compact Hausdorff spaces may skip §§ 11–14. Section 15 is absolutely essential and should be read with careful attention. Sections 16–18 may be omitted. Sections 19 through 24 are vital, and should be read carefully. Sections 25 and 26 are rather special, but are so interesting that we hope all readers will work through them.

Sections 4–11 and 15–26 contain subsections entitled Miscellaneous Theorems and Examples. Some of these are worked out in detail; for others, proofs are sketched or omitted entirely. We refer *occasionally* in the main text to a result drawn from the Miscellaneous Theorems and Examples. All such results are easy and are supplied with proofs. The reader is counselled at least to read the statements of the Miscellaneous Theorems and Examples, and to use them as exercises *ad libitum*.

Many sections also contain historical notes. We have tried to trace the history of the principal theorems and concepts, but we obviously have not produced a complete history, and it would be foolish to claim that we have produced one correct in every detail. Also, while some of the results we give are new so far as we know, *failure to cite a reference for a given theorem should not be construed as a claim of originality on our part*.

For the reader's convenience, we have assembled in three appendices certain ancillary material not easily accessible elsewhere, which is essential for one part or another of the theory. These appendices may be read as the reader encounters references to them in the main text.

We have been assisted by many colleagues. W. WISTAR COMFORT, JOHN M. ERDMAN, ALBERT J. FRODERBERG, L. FUCHS, JAMES MICHELOW, RICHARD S. PIERCE, KARL R. STROMBERG, ELMAR THOMA, VEERAVALLI S. VARADARAJAN, and HERBERT S. ZUCKERMAN have all read various parts of the manuscript, made valuable improvements, and saved us on occasion from grievous error. Improvements have also been made by NORMAN J. BLOCH, FRANCIS X. CONNOLLY, GERALD L. ITZKOWITZ, and RICHARD T. SHANNON. For the help so generously given by our friends we are sincerely grateful. Our thanks are due to Milles. LYNNE HARPER and JEANNE SLOPER for preparation of the typescript. Mr. ALBERT J. FRODERBERG has most generously assisted us in the task of proof-reading.

The reading and research by both of us on which the book is based have been generously supported by the National Science Foundation, U.S.A., and by a fellowship of the John Simon Guggenheim Memorial Foundation granted to E. HEWITT. We extend our sincere thanks to the authorities of these foundations, without whose aid our work could not have been done.

Finally, we record our gratitude to Prof. Dr. F.K. SCHMIDT for his original suggestion that the book be written, and to the publishers for their dispatch, skill, and attention to our every wish in producing the book.

Seattle, Washington,
Rochester, New York

EDWIN HEWITT
KENNETH A. ROSS

October 1962

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