A Monte Carlo Primer

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A Practical Approach to Radiation Transport

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To Pat and Huri

Preface

The mathematical technique of Monte Carlo, as applied to the transport of sub-atomic particles, has been described in numerous reports and books since its formal development in the 1940s. Most of these instructional efforts have been directed either at the mathematical basis of the technique or at its practical application as embodied in the several large, formal computer codes available for performing Monte Carlo transport calculations. This book attempts to fill what appears to be a gap in this Monte Carlo literature between the mathematics and the software. Thus, while the mathematical basis for Monte Carlo transport is covered in some detail, emphasis is placed on the application of the technique to the solution of practical radiation transport problems. This is done by using the PC as the basic teaching tool.

This book assumes the reader has a knowledge of integral calculus, neutron transport theory, and Fortran programming. It also assumes the reader has available a PC with a Fortran compiler. Any PC of reasonable size should be adequate to reproduce the examples or solve the exercises contained herein. The authors believe it is important for the reader to execute these examples and exercises, and by doing so to become accomplished at preparing appropriate software for solving radiation transport problems using Monte Carlo. The step from the software described in this book to the use of production Monte Carlo codes should be straightforward. However, instead of using these production codes as “black boxes” that must be taken on faith, the reader should be able to understand the functions being performed by the various components of the production software, and in most cases be capable of modifying or expanding it to suit the needs of a particular calculation.

Mathematical procedures can be expressed in many different ways using high-level computer languages. Fortran is no exception to this generality. Thus there is no “best” way to write a Fortran procedure, and many individual styles, all of which can produce equally valid results, are possible. In our examples we employ executable statements based on Fortran-90, but have often retained the style of older versions of Fortran. We have done this because of our particular experience in using various versions of Fortran. Hopefully readers will be able to follow the constructs easily and to re-cast them into a form compatible with their preferences. To save space we have frequently placed several executable statements in a line of coding.

We have adopted several conventions in the presentation of the text and examples. Fortran subroutines and variable names are designated by single quotation marks. Numerical exponents are expressed either by using the base 10 with a superscript or using the convention of ‘+’ or ‘-’ to designate the exponent; i.e., $1.0+5 = 1.0E+5 = 1.0 \times 10^5$. Tabulated results are rounded, with the number of significant digits shown being that which seems appropriate for the example in hand. All calculations that involve using computed results to obtain additional results use the values prior to rounding. Therefore, if the derived results are reproduced using the rounded values presented in the tables the numbers obtained may differ from those cited in the text. Because compilers and execution speeds differ, the user should not expect to reproduce the problem run times cited in this book. However, the relative efficiencies of a series of calculations should be similar regardless of the compiler and computer used.

We gratefully acknowledge the assistance of our friends and colleagues who reviewed the manuscript. Kevin O’Brien and Jim Renken provided thorough and detailed reviews of the entire text, and Eleanor Walther reviewed the appendix. In all cases their comments were well considered and informative, and they saved us from numerous omissions, obfuscations, and errors. Special thanks go to Len Connell and Tom Laub, who not only reviewed the text but also devoted considerable effort to checking and using the Monte Carlo routines. Long before the manuscript was completed Len Connell was the first user of the probabilistic framework code. Tom Laub provided independent solutions and numerous corrections to the example problems. The time and efforts of these reviewers are much appreciated and this book has greatly benefited from their support. Needless to say, any errors remaining in the text are the responsibility of the authors.

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