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Ben-Zion Maytal · John M. Pfotenhauer

Miniature Joule-Thomson Cryocooling

Principles and Practice

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to my wife ***Hana***
Ben-Zion Maytal

to my wife ***Nadine***
John Pfotenhauer

Preface

This book is the first in English, being entirely dedicated to the topic of *Miniature Joule-Thomson Cryocooling*. The sole previous book on the same subject, *Throttle Type Microrefrigerators*, was published in Russian in Moscow 35 years ago, by Suslov et al. Various books on the general topic of cryogenics and even those focused on cryocoolers include at most not more than a single chapter on Miniature Joule-Thomson cryocooling. The authors of this book have been motivated by the conviction that the subject deserves an updated, broader and deeper treatise.

The five parts of the present book include nine chapters, arranged according to a detailed list of content, and an index.

Part I, comprising Chap. 1, attempts to portray all cryocoolers from a common basis and focuses on the uniqueness of miniature Joule-Thomson cryocooling from this perspective. In fact, the common basis can aptly serve as a first and preliminary lesson whenever one introduces the subject of cryocoolers.

Part II combines Chaps. 2, 3, and 4, to comprise the *theoretical foundation* of the subject. Chapter 2 focuses on the Joule-Thomson effect in both differential and integral forms, as well as the inversion of the same effect. Chapter 3 discusses the thermodynamic principles behind the Linde-Hampson liquefaction process. Chapter 4 identifies the *real gas properties* that are key parameters of candidate coolants for Joule-Thomson cryocooling. The deviation of "real gas" properties from the ideal gas model is an alternative expression of the Joule-Thomson effect that drives the Joule-Thomson cryocooling process.

Part III includes Chaps. 5, 6, and 7 and deals with the *practical* and *application aspects* of the subject. Chapter 5 scans the variety of operating modes including continuous, staged, fast cooldown, and hybrid. Chapter 6 focuses strictly on details of construction and configuration. Chapter 7 deals thoroughly with aspects of the transient behavior during the cooldown of Joule-Thomson cryocoolers. However, cooldown is also discussed in Chap. 5 in the context of the fast cooldown mode of cryocooling.

Parts II and III of the book correspond with the traditional process for pure gases. These sections address Joule-Thomson cryocooling in the manner used by *Linde* and *Hampson*, but focuses on the small-scale, miniature version operated in an open cycle and at elevated pressure.

Part IV, which is Chap. 8, is entirely dedicated to the use of *mixed coolants* for Joule-Thomson cryocooling. Mixed coolants enable new possibilities that are not attainable with pure coolants. Chapter 8 highlights both the theoretical topics and the practical issues for this type of Joule-Thomson cryocooler, either with or without phase separators, and therefore it is bigger than the others.

Part V, the ninth and last chapter, gathers various *special topics* of general significance and relevance that do not fit well under the headings of the first eight chapters, such as gas purity, choked flow rates of real gases, modeling of cryocoolers, cryosurgical devices, and warming via the Joule-Thomson effect.

The field of cryocoolers as a branch of cryogenics is continuously growing and developing. Joule-Thomson cryocooling, defined by the Linde-Hampson process, has a special position within this group. It uniquely depends upon the real gas properties of the coolants, that is, their deviation from the ideal gas model. These aspects may attract not only people who are directly involved in miniature Joule-Thomson cryocooling but also those possessing a general interest in the disciplines of thermodynamics and cryogenics.

A detailed list of references, chapter by chapter, provides a broad literature survey; it consists of more than 1,200 relevant articles in addition to more than 450 related *patents*. Patents expose a variety of ideas and practical engineering experience, and therefore frequently unfold important details of construction.

Various topics are explored in a *chronological perspective* (such as the inversion of the Joule-Thomson effect, the integral inversion curve, mixed gas cryocooling, and flow regulating mechanisms).

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Nomenclature

a	Attraction parameter in van der Waals' equation of state, N mole^{-1}
	Cross sectional area (of an orifice), m^2
$\bar{a}$	Normalized area of an orifice in a closed cycle cryocooler, Eq 8.22
A	Area, m^2
b	Repulsion parameters in van der Waals' equation of state, $\text{m}^3\ \text{mol}^{-1}$
B, C, D	Second, third, fourth etc. virial coefficient of an equation of state expanded by density, Appendix A2.3
$B',\ C', D'$	Second, third, fourth etc. virial coefficient of an equation of state expanded by pressure, Appendix A2.3
c	Speed of sound, m s^{-1}
	Elasticity coefficient (of a spring), N m^{-1}
	Concentration, mol m^{-3}
c_P	Isobaric specific heat capacity, J $\text{mol}^{-1}\ \text{K}^{-1}$
c_V	Isochoric specific heat capacity, J $\text{mol}^{-1}\ \text{K}^{-1}$
c_{PO}	Isobaric specific heat capacity at zero pressure, J $\text{mol}^{-1}\ \text{K}^{-1}$
c_{V0}	Isochoric specific heat capacity at zero pressure, J $\text{mol}^{-1}\ \text{K}^{-1}$
C	Total heat capacity, J
	Cost of cooling or of liquefying a cryogen, J/J or J mol^{-1}
$\dot{C}$	Capacity rate, $\dot{n}\ c_P$, J s^{-1}
COP	Coefficient of performance, Eq 1.74
D, d	Outer and inner diameters, m
d	Volumetric displacement of a compressor, swept volume, m^3
d^{HYD}	Hydraulic diameter, $4\,A/p$
d_{HELIX}	Diameter of the helix of a coiled tube, m
f	Fanning friction factor, Eqs 3.160 and 3.161
	Frequency, s^{-1}
F	Force, N
FOM	Figure of merit, $\text{COP}/\text{COP}_{\text{CARNOT}}$
G	Mass flux, $\dot{n}/A$, mol $\text{m}^{-2}\ \text{s}^{-1}$
g	Gravitational acceleration, 9.81 m s^{-2}
	Gibbs free energy, $h - T\ s$
h	Specific enthalpy, J mole^{-1}
	Convective heat transfer coefficient, W $\text{m}^{-2}\ \text{K}^{-1}$
h^0	Specific stagnation enthalpy, $h + \rho\ V^2/2$, J mol^{-1}
h_G	Specific enthalpy of saturated gas, J mol^{-1}

h_F	Specific enthalpy of saturated liquid, J mol^{-1}
h^R	Residual specific enthalpy, based on common $(P,\ T)$, J mol^{-1}
h^{IG}	Ideal gas specific enthalpy, J mole^{-1}
Δh_T	Specific enthalpy change at constant temperate, (integral isothermal Joule-Thomson effect), specific cooling content of gas, J mol^{-1}
Δh_T^{MAX}	Maximum Δh_T value along the differential inversion curve, J mol^{-1}
h^{MIX}	Specific enthalpy of a mixture, J mol^{-1}
Δh^{MIX}	Specific mixing enthalpy, J mole^{-1}
δh_D	Downstream recuperated enthalpy, $h_5 - h_4$, J mol^{-1}
δh_U	Up stream recuperated enthalpy, $h_2 - h_1$, J mol^{-1}
H	Total enthalpy, J
i	Ineffectiveness of a heat exchanger, $1 - \varepsilon$
I	Irreversibility, the loss of potential useful work, J
	Inefficiency, $1 - \eta$
I_M	Interchanger's magnification index (Sect. 1.4)
k	Permeability of a porous media, Eq. 6.9, m^2
	Boltzmann's constant, R/N, J K^{-1}
K_1	Discharge coefficient, $\dot{n}/P$, Eq. 9.12, mol s^{-1} Pa^{-1}
K_2	Discharge coefficient, $\dot{n}\big/\sqrt{P}$, Eq. 9.13, mol s^{-1} $\mathrm{Pa}^{-1/2}$
K_T	Isothermal compressibility, $\rho^{-1}\ (\partial\rho/\partial P)_T$ or $-v^{-1}\ (\partial v/\partial P)_T$, Pa^{-1}
L	Latent heat of evaporation, $h_G - h_F$, J mole^{-1}
L^*	Critical length for reaching speed of sound, m
l	Length, m
m	Mass, kg
$\dot{m}$	Mass flow rate, recuperated flow rate of a cryocooler, kg s^{-1}
M	Molar mass, kg mol^{-1}
Ma	Mach number, V/c
n	Number of moles
$\dot{n}$	Molar flow rate, mole s^{-1}
n_{LIQ}	The mass of liquefied coolant in the cryocooler, mol
$\dot{n}_{FR}$	Warm ("free") non recuperated molar flow rate, mol s^{-1}
$\dot{n}_{RE}$	Recuperated flow rate of a cryocooler, mole s^{-1}
N	Avogadro's number (of molecules in a single mole), $6.023 \cdot 10^{23}$
NTU	Number of Heat Transfer Units of a heat exchanger, $U\,A\Big/\dot{C}^{MIN}$
Nu	Nusselt Number, $h\ d/\lambda$
p	Perimeter, m
P	Pressure, Pa
P_C	Critical pressure, Pa
P_S	Saturation pressure, Pa
P_{TR}	Triple point pressure, Pa
P^*	Regulated pressure, Pa
q	Heat flux, W m^{-2}
Q	Heat, J
Q_V	Volumetric cooling content (per unit volume of displaced coolant), $\dot{Q}\big/\dot{V}$, J m^{-3}
$\dot{Q}$	Cooling capacity or heat load at steady state operation, W
	Rate of heat transfer, W
$\dot{Q}_{LOAD}$	Heat load, W

r	Capacity rate ratio of heat exchanging streams, $\dot{C}^{MIN} \big/ \dot{C}^{MAX} \leq 1$
R	Universal gas constant, 8.314 J mol^{-1} K^{-1}
Re	Reynolds number, $V\ d/\upsilon$

s	Specific entropy, J mole^{-1} K^{-1}
s^r	Specific residual entropy based on common $(\rho,\ T)$, J mol^{-1} K^{-1}
S	Total entropy, J K^{-1}
	Surface area parallel to the direction of flow, m^2
SP	Splitting ratio of a liquefier, Sect. 3.8.2.3

t	Time, s
t_{CD}	Cooldown period, s
T	Absolute temperature, K
T_{BOIL}	Normal boiling temperature, K
T_{BOYLE}	Temperature of Boyle, satisfying $B(T) = 0$, K
T_C	Critical temperature, K
T_{CD}	Cooldown temperature from which an isenthalpic expansion generates the first liquid yield (Sects. 3.2.1.6 and 4.4.1), K
T_D	Temperature of the state where the differential inversion curve intersects the saturation line, K, (Sect. 2.4.4)
T_{EU}	Temperature associated with eutectic composition, K
T_{INV}	Inversion temperature of the Joule-Thomson effect, K
T_M	Normal melting temperature, K
TND	Cooldown tendency of gases, $\lvert \varDelta T_h \rvert / \varDelta T_{CD}$, (Sect. 7.4.4)
T_2, T_3	Temperatures at the inlet and outlet of the Joule-Thomson valve of a Linde-Hampson device , K
T_4	Temperature at the (returning) inlet to recuperator's low pressure channel of a Linde-Hampson device, K
T_{PINCH}	Pinch point temperature of a heat exchanger, K
T_{PRE}	Precooling temperature of a final stage, K
T_{TR}	Triple point temperature, K
T_S	Saturation temperature, K
T_λ	Lambda point, super fluidity temperature of transition, K
$\varDelta T$	Temperature difference (between the two streams of the heat exchanger), K
$\varDelta T_{CD}$	Cool down temperature range, $T_{AMB} - T_{CD}$, Sect. 4.4.1, K
$\varDelta T_h$	Integral isenthalpic Joule-Thomson effect, K
$\varDelta T_{LMTD}$	Logarithmic mean temperature difference, Eq. 3.92, K

u	Internal energy, J
U	Total heat transfer coefficient, W m^{-2} K^{-1}
	Total internal energy, J
UA	Thermal conductance between the channels of a counter flow heat exchanger, W K^{-1}

V	Velocity, m s^{-1}
	Volume, m^3
$\dot{V}$	Rate of volumetric displacement (of a compressor), m^3 s^{-1}
v	Specific volume, $1/\rho$, m^3 mol^{-1}
v^R	Residual volume, $v - v^{IG}$, m^3 mol^{-1}

w	Specific loss of availability due to irreversibility, I/n, J mol^{-1}
W	Work, J
W^{LOST}	Work or availability loss associated with irreversibility, J

x	Axial longitudinal position coordinate along the helical finned tube heat exchanger, m
	Quality of a two phase fluid, mass fraction of the vapor phase
X	Longitudinal coordinate along the heat exchanger, m
	Composition, molar fraction
y	Liquefaction yield, mass fraction of a stream
Y	Relative compressibility, $R\ T\ (\partial\rho/\partial P)_T$, Sect. 4.7
z	Molar fraction of a component in a mixture
Z	Compressibility, $P\ v/(R\ T)$

Greek Symbols

α	Thermal diffusivity, $\lambda/(\rho\ c_P)$, m^2 s^{-1}
	Coefficient of thermal expansion, $\Delta l/(l \cdot \Delta T)$, K^{-1}
α_C	Acentricity factor of Riedel, Eq. A2.35
β	Thermal expansivity, $-\rho^{-1}\ (\partial\rho/\partial T)_P$ or $v^{-1}\ (\partial v/\partial T)_P$
ε	Effectiveness of a heat exchanger
	Eddy viscosity (of turbulent flow), N s m^{-2}
Φ	Reduced specific density, ρ/ρ_C
	Dissipation function, Sect. 3.12.2
Γ	The ratio of real gas choked mass flux and the ideal gas, G/G^{IG}
ϕ	Isothermal Joule-Thomson coefficient, $(\partial h/\partial P)_T$, m^3 mole^{-1}
η	Efficiency of a heat exchanger
	Joule’s free expansion coefficient by pressure, $(\partial T/\partial P)_u$, K Pa^{-1}
η^*	Joule’s free expansion coefficient by volume, $(\partial T/\partial v)_u$, K mol m^{-3}
η_{EX}	The efficiency of the isentropic expander
η_f	Thermal efficiency of a fin
η_V	Volumetric efficiency of a compressor
η_I	First Law efficiency
η_{II}	Second Law efficiency, exergy efficiency
θ	Ratio of low and high counter flow inlet temperatures, $T_{L,IN}/T_{H,IN}$
κ	Specific heat ratio, c_P/c_V
λ	Thermal conductivity, W m^{-1} K^{-1}
μ	Adiabatic Joule-Thomson coefficient, $(\partial T/\partial P)_h$, K Pa^{-1}
	Dynamic viscosity, N s m^{-2}
	Chemical potential, J mol^{-1}
ν	Kinematic viscosity, μ/ρ, m^2 s^{-1}
Π	Reduced pressure, P/P_C
ρ	Density, $1/v$, mol m^{-3}
τ	Time constant, thermal or hydrodynamic delay, s
	Shear stress, Pa
Θ	Reduced temperature, T/T_C
Σ	Scaling parameter of a recuperator, $\delta h/\Delta h_T$, Sect. 3.7.6
ω	Acentricity factor of Pitzer, Eq. A2.14
ξ	Fraction of maximum integral isothermal Joule-Thomson effect, $\Delta h_T/\Delta h_T^{MAX}$, Sect. 4.3.7.2
ζ	Satellite cooler injected mass fraction, Sect. 3.1.2, Fig. 3.1

Subscripts

0	Zero pressure Stagnation conditions Initial
1, 2, 3, 4, 5	Thermodynamic states at the generalized model of cryocoolers in Fig. 1.2 and the Linde-Hampson cycle in Figs. 3.3, 3.4, and 3.5
AMB	Ambient
BOIL	Boiling
C	Critical state
CARNOT	Ideal reversible Carnot cycle
CD	Cooldown
CUT	Cut off pressure
C.V.	Control volume
D	Downstream, low pressure expanded stream of a counter flow heat exchanger
DEWAR	Dewar
EFF	Effective value
EVP	Evaporator
EU	Eutectic composition
F	Saturated liquid state Final
G	State of saturated gas
H	High, temperature or pressure, hot stream
HE	Heat exchanger
HELIX	Helix
IN	Inlet stream
INV	Joule-Thomson inversion state
JT	Joule-Thomson
L	Low, temperature or pressure, cold stream
OPT	Optimum
OUT	Outlet stream
PRE	Pre-cooling
PINCH	Pinch point along the heat exchanger
S	Saturation conditions
ST	Standard conditions
REV	Reversible
TUBE	Tube
TR	Triple point
U	Up stream, high pressure incoming stream of a counter flow heat exchanger

Superscripts

(1)	Cooler
(2)	Liquefier
(A)	Referring to component A of a the mixture
i	The i-th component of a mixture The i-th stage of a staged cryocooler

CD	Cooldown
HYD	Hydraulic
IG	Ideal gas
MIN	Minimum value
MAX	Maximum value
MIX	Referring to the mixture
MIXING	Referring to the mixing process
r	Residual property, deviation from the ideal gas at the common (ρ, T)
R	Residual property, deviation from the ideal gas at the common (P, T)
REF	Reference value
SS	Steady state

Abbreviations

AC	Alternating current, referred to pulsating flow
CD	Cooldown
CFD	Computational Fluid Dynamics
C.V.	Control volume
DC	Direct current, referred to continuous flow
D.I.C.	Differential inversion curve
EOS	Equation of state
GRS	Gas refrigerant supply
I.I.C.	Integral inversion curve
JT	or J-T Joule-Thomson
Lit	Liter
LRS	Liquid refrigerant supply
MTBF	Mean time between failures
NG	Natural gas
PPM	Parts per million, by volume
SLPM	Standard litters per minute
μm	Micro meter as a measure of length, 10^{-6} m

General Notation

$[X]$	The unit of the entity X; for instance, $[F]$=N
$\equiv$	Identity, used for definition
$\bar{X}$	Average of the variable X
$\overline{\mathrm{AB}}$	Length of a section starting at point A and ending at B
$\dot{X}$	Time derivative of X
$\sigma(X)$	Standard deviation (variance) of the distribution of the variable (X)
$\left(\frac{\partial y}{\partial x}\right)_z$	Partial derivative of the variable y by x while preserving z