

Lecture Notes in Physics

Volume 835

Founding Editors

W. Beiglböck
J. Ehlers
K. Hepp
H. Weidenmüller

Editorial Board

B.-G. Englert, Singapore
U. Frisch, Nice, France
F. Guinea, Madrid, Spain
P. Hänggi, Augsburg, Germany
W. Hillebrandt, Garching, Germany
M. Hjorth-Jensen, Oslo, Norway
R. A. L. Jones, Sheffield, UK
H. v. Löhneysen, Karlsruhe, Germany
M. S. Longair, Cambridge, UK
M. Mangano, Geneva, Switzerland
J.-F. Pinton, Lyon, France
J.-M. Raimond, Paris, France
A. Rubio, Donostia, San Sebastian, Spain
M. Salmhofer, Heidelberg, Germany
D. Sornette, Zurich, Switzerland
S. Theisen, Potsdam, Germany
D. Vollhardt, Augsburg, Germany
W. Weise, Garching, Germany

For further volumes:

<http://www.springer.com/series/5304>

The Lecture Notes in Physics

The series Lecture Notes in Physics (LNP), founded in 1969, reports new developments in physics research and teaching—quickly and informally, but with a high quality and the explicit aim to summarize and communicate current knowledge in an accessible way. Books published in this series are conceived as bridging material between advanced graduate textbooks and the forefront of research and to serve three purposes:

- to be a compact and modern up-to-date source of reference on a well-defined topic
- to serve as an accessible introduction to the field to postgraduate students and nonspecialist researchers from related areas
- to be a source of advanced teaching material for specialized seminars, courses and schools

Both monographs and multi-author volumes will be considered for publication. Edited volumes should, however, consist of a very limited number of contributions only. Proceedings will not be considered for LNP.

Volumes published in LNP are disseminated both in print and in electronic formats, the electronic archive being available at springerlink.com. The series content is indexed, abstracted and referenced by many abstracting and information services, bibliographic networks, subscription agencies, library networks, and consortia.

Proposals should be sent to a member of the Editorial Board, or directly to the managing editor at Springer:

Christian Caron
Springer Heidelberg
Physics Editorial Department I
Tiergartenstrasse 17
69121 Heidelberg/Germany
christian.caron@springer.com

Martin Bojowald

Quantum Cosmology

A Fundamental Description of the Universe

Prof. Dr. Martin Bojowald
Institute for Gravitation and the Cosmos
Pennsylvania State University
104 Davey Laboratory
University Park, PA 16802-6300
USA
e-mail: bojowald@gravity.psu.edu

ISSN 0075-8450

ISBN 978-1-4419-8275-9

DOI 10.1007/978-1-4419-8276-6

Springer New York Dordrecht Heidelberg London

e-ISSN 1616-6361

e-ISBN 978-1-4419-8276-6

© Springer Science+Business Media, LLC 2011

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC, 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Cover design: eStudio Calamar, Berlin/Figueres

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Contents

1	Introduction	1
Part I Quantizing the Whole Universe		
2	Cosmology and Quantum Theory	7
2.1	Scaling	8
2.2	Wheeler–DeWitt Quantization	9
2.3	Evolution	10
2.4	Bohmian Viewpoint	11
2.5	WKB Approximation	12
2.6	General Problems	14
	References	15
3	Kinematics: Spatial Atoms	17
3.1	Quantized Particles	17
3.2	Quantized Space–Time	22
3.2.1	Scaling	23
3.2.2	Canonical Gravity	24
3.2.3	Isotropic Models	33
	References	45
4	Dynamics: Changing Atoms of Space–Time	47
4.1	Refinement and Internal Time	47
4.2	Constructions in the Full Theory	49
4.2.1	Gravitational Constraint	49
4.2.2	Matter Hamiltonian	52
4.2.3	Problem of Dynamics	54
4.3	Isotropic Universes	54
4.3.1	Symmetry	55
4.3.2	Models	56
4.3.3	Quantum–Geometry Corrections	58

4.4	The Role of an Underlying State	60
4.4.1	Refinement Models	60
4.4.2	Interpretations.	62
4.4.3	Refinement from Reduction	64
4.5	Basic Singularity Removal: Quantum Hyperbolicity	66
	References	68
 Part II Effective Descriptions		
5	Effective Equations	73
5.1	Quantum Effects in Separation	73
5.2	Wave-Function Dynamics.	76
5.3	Solvable Models for Cosmology	77
5.3.1	Wheeler–DeWitt Quantization	77
5.3.2	Loop Quantization.	82
5.4	Isotropic Perturbation Theory: Spatial Curvature, a Cosmological Constant, and Interacting Matter	84
5.4.1	Free Matter	84
5.4.2	Scalar Potential.	95
	References	96
6	Harmonic Cosmology: The Universe Before the Big Bang and How Much We Can Know About It	99
6.1	Reality	99
6.2	Uncertainty.	101
6.3	Repulsive Forces and Bouncing Cosmologies	102
6.4	Quantum Big Bang	106
6.5	Dynamical Coherent States	110
6.6	Lessons for Effective Actions	116
	References	117
7	What Does It Mean for a Singularity to be Resolved?	119
7.1	Density Bounds	119
7.2	Bounces	120
7.3	Quantum Cosmology	124
7.3.1	Interpretational Issues	125
7.3.2	Examples	125
7.3.3	Dependence on Ambiguities.	126
7.4	Negative Attitude	127
	References	128

Part III Beyond Isotropic Models

8 Anisotropy 133

 8.1 Constructing Models 133

 8.2 Bianchi Cosmology 136

 8.2.1 Bianchi Dynamics 136

 8.2.2 Connection Variables and Holonomies 138

 8.2.3 Dynamics and Refinement 143

 8.2.4 Reduction from Anisotropy to Isotropy 146

 8.3 Black Hole Models Inside the Horizon. 157

 8.3.1 Canonical Formulation. 158

 8.3.2 Loop Quantization. 159

 8.3.3 Tree-Level Equations 162

 8.3.4 Lattice Refinement 162

 8.3.5 Singularity 164

 References 164

9 Midisuperspace Models: Black Hole Collapse 167

 9.1 Spherical Symmetry. 167

 9.1.1 Canonical Transformation 170

 9.1.2 States 172

 9.1.3 Basic Representation from the Full Theory 175

 9.1.4 Hamiltonian Constraint 178

 9.1.5 Lemaître–Tolman–Bondi Models and Gravitational
 Collapse. 180

 9.1.6 Further Applications of Spherical Symmetry 182

 9.2 Models with Local Degrees of Freedom. 186

 9.2.1 Models 187

 9.2.2 Connection Formulation. 188

 9.2.3 Hybrid Quantization 190

 9.3 Properties 191

 9.3.1 Non-Singular Behavior 191

 9.3.2 Lattice Refinement and Anomaly Freedom 193

 References 194

10 Perturbative Inhomogeneities 197

 10.1 Formalism 197

 10.1.1 Linear Modes in Cosmological Perturbations 197

 10.1.2 Basic Operators 199

 10.1.3 Composite Operators 206

 10.1.4 Quantum Corrections. 209

 10.2 Quantum Corrections in Effective Equations. 220

10.2.1	Holonomy Corrections	221
10.2.2	Inverse-Triad Corrections	221
10.2.3	Types of Corrections	224
10.3	Anomaly-Freedom of Quantum Corrections	225
10.3.1	Consistency Issues	225
10.3.2	Quantum Corrections	230
10.3.3	Scalar Modes	233
10.3.4	Vector Modes	234
10.3.5	Gravitational Waves	236
10.3.6	Comparison with Gauge-Fixed Treatments	237
10.4	Cosmological Applications	238
10.4.1	Quantum-to-Classical Transition	238
10.4.2	Big-Bang Nucleosynthesis	239
10.4.3	Super-Inflation	239
10.4.4	Scalar Modes	240
10.4.5	Tensor Modes	241
10.4.6	Indications for Evolution Through a Bounce	241
	References	243

Part IV Mathematical Issues

11	Difference Equations	247
11.1	Singularities and Dynamical Initial Conditions	247
11.2	Properties	251
11.2.1	Stability	251
11.2.2	Boundedness	260
11.3	Non-Linear Loop Quantum Cosmology	262
	References	263
12	Physical Hilbert Spaces	265
12.1	Group Averaging	265
12.2	Observables	268
12.3	Internal Time	269
12.3.1	Non-Relativistic Parameterized Systems	269
12.3.2	Relativistic Parameterized Systems	270
12.4	Examples in Quantum Cosmology	272
	References	274
13	General Aspects of Effective Descriptions	275
13.1	Canonical Effective Equations	275
13.1.1	States and Moments	276
13.1.2	Quantum Phase Space	276
13.1.3	Relations	277
13.1.4	Casimir Conditions	280

- 13.1.5 Equations of Motion 280
- 13.1.6 General Properties. 287
- 13.2 Effective Constraints 290
 - 13.2.1 Non-Relativistic Constraints 291
 - 13.2.2 Relativistic Systems 295
 - 13.2.3 Problem of Time. 295
- 13.3 Applications of Effective Constraints in Quantum Gravity and
Cosmology 297
 - 13.3.1 Problem of Time. 298
 - 13.3.2 Anomaly Problem 298
- References 299

- 14 Outlook 301**

- Index 303**