

# Fundamental Theories of Physics

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Tamás Sándor Biró

# Is There a Temperature?

Conceptual Challenges at High Energy,  
Acceleration and Complexity

 Springer

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*To Tünde, Szilvia, Réka, Gábor and Kati.*



# Preface

Yet another book about thermodynamics? Wherefore? This almost 200-year-old physics subject is being discussed in depth and presented in a plethora of textbooks extensively. It is not my intention to add one to this number. It is the experience in modern research problems dealing with matter under extreme conditions which challenges me to write a book about this topic. Physical matter at high energy, velocity and momenta, under extreme acceleration or deceleration, or by another particular circumstance, such as unusual complexity, does behave strangely. Talking about extremely high or low, by the familiar thermometer not measurable temperatures, leads us not only to the question, what is the temperature of such matter, but also to the question, whether the very concept of absolute temperature is applicable: Is there a temperature at all?

The more important this question becomes since the “social” disciplines of physics, like biological physics, econophysics, sociological models started to apply mathematical models and concepts originally devised for the study of “ordinary,” i.e. atomic matter. These generalized statistical and stochastic models apply quantities analogous to the physical energy, to the entropic measure of order and disorder, and to the associated concept of absolute temperature. By such applications of physics, however, the exception seems to be the rule: most studied distribution properties are peculiar from the classical thermodynamics viewpoint. To give an example, the Black–Sholes model, describing pricing strategies for derivative financial transactions,<sup>1</sup> is mathematically a classical diffusion model, like the Fokker–Planck equation. Meanwhile, it turned out that more realistic models describing the price fluctuations do show a fat tail, namely a non-Gaussian distribution. Such tails, frequently occurring as power-law tails, can be described as a result of anomalous diffusion. The general theory dealing with such distributions, as classical thermodynamics does with the canonical exponential energy distribution, is in its making currently. One of her tentative names is *non-extensive thermodynamics*.

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<sup>1</sup> The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel was awarded to Robert C. Merton and Myron S. Sholes in 1997, who worked out this model in close collaboration with Fisher Black.

However, this book is not another introduction into non-extensive thermodynamics either. There are good books devoted to this task. The intention here is to present basic concepts at the very heart of statistical physics as they are challenged in high energy nuclear and particle physics by phenomena like energy distribution of particles irradiated from a fireball mimicking the Big Bang in little, like a quite formal use of absolute temperature as a parameter of higher than four-dimensional objects or like the particular behavior of colored noise in the dynamics of elementary fields. By doing so we particularly concentrate to the recurring question whether all the “anomalous” thermodynamics behavior is just a finite-size finite-time effect, or it survives the (in several cases only theoretical) limit of a large number of degree of freedom, commonly called the “thermodynamical limit.” This question can presently be answered in the mathematical framework; the answers in physical experiments seem to be delegated to the future. Notably, also the question: Is there a temperature? in these exotic, highly energetic physical phenomena.

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*Tamás Sándor Bíró*



# Acknowledgments

There are due acknowledgments for a number of persons. The most important catalyzing event to start writing this book was a general question on the  $\Sigma\Phi$  (Statistics in Physics) Conference, in Kolymbari, Crete, in 2008 from an Editor of Springer Verlag to the participants: whether anyone had an idea to write a good, interesting physics book. I attended that conference and subconsciously began thinking about how to confront some uncleared usage of statistical physics methods in the area of high-energy physics, more closely heavy-ion collisions, with challenging questions of contemporary statistical physics – which the conference was partially organized about. I responded to the inquiry that I possibly would have an idea.

One year later there came an e-mail from Jeanine Burke, of Springer Verlag, enquiring if I still had the idea to write a book about challenges at the merge of high-energy and statistical physics. In fact, I was already occupied with the writing of another book, about variational principles in the basic laws of physics, at that time. Nevertheless, I responded positively and sent an outline of the contents, a quickly written introduction, and filled out a detailed questionnaire to the Editors. They then selected this idea to be a project, we signed a contract, and so I began work on it. I have to thank the Springer Verlag for motivating me to really work on this project, and to Rebecca Nison for escorting my authorship.

From my colleagues in Physics I thank first of all Prof. Constantino Tsallis, who – besides being a well-known promoter of the new view on the possible extension of thermodynamics and statistical physics beyond the classical grounds – also personally supported my efforts to understand and generalize the classical concepts in my own way and to present part of the problems from my particular corner of view in several discussions. Dr. Giorgio Torrieri from the Frankfurt University helped me a lot on getting into contact with works on classical and string-theory induced higher-dimensional gravity and this way to put the perspective on really far-stretched theoretical challenges on the concepts of temperature and entropy. Furthermore, I thank Prof. Miklós Gyulassy from Columbia University for his patient conversations with me and for his critical remarks about this topic. I have learned a lot about which questions would be important to answer.

There are more colleagues with whom I had the privilege to co-author scientific publications, whose results are in one or the other way now involved in the material discussed in this book. I thank them for their inspiration and help. With Prof. Rogerio Rosenfeld I had my adventure on the field of financial models and learned the importance of coupled stochastic dynamics in challenging the classical models. Prof. Berndt Müller from the Duke University involved me in the research on the role of chaotic dynamics in non-Abelian field theory and – developing from this study – on possible spontaneous mechanisms for an emerging quantum theory from a higher-dimensional, two-time variable background. This settled the fundamentals of my interest in questioning beyond the “textbook wisdom”. With Prof. Carsten Greiner (now at Frankfurt University, that time at Duke University and at Giessen University) I have started studies on stochastic dynamics in field theory in connection with the – there common – graph-language of Feynman and Keldysh. This paved my way to become interested in the role of noise on the most elementary level in physics.

Important parts of this book are rooted in common research with the following colleagues. The study with Dr. Jakovác on multiplicative noise and the production of power-law tailed energy distributions in stochastic field theory applied to elementary particle plasmas was induced by the challenge of non-extensive behavior in systems with long range micro-dynamical effects. In the thermodynamical interpretation and in selecting out far-reaching consequences of observations on power-law tailed particle spectra in heavy-ion collisions, Prof. Géza Györgyi from the Eötvös University in Budapest was an excellent discussion partner and a wealthy source of original ideas. The study and computer modeling of those spectra was (and to some extent is) carried out in collaboration with my younger colleagues Gábor Purcsel, Dr. Gergely Barnaföldi, and Károly Ürmösy at my own institute, the KFKI RMKI, Budapest. Last but not the least, Dr. Péter Ván himself having a long history of personal interest in fundamental questions of thermodynamics, worked with me on deepening and refining our understanding about relativistic thermodynamics, in particular on its possible connections to theoretical practices in contemporary applications of relativistic hydrodynamics to the description of high-energy particle systems.

Finally, I would like to thank especially my colleagues Dr. Antal Jakovác from the Budapest Technical University, Dr. István Rácz, Dr. Etele Molnár, and Dr. Péter Ván from KFKI RMKI research institute in Budapest for their countless discussions and active support during the preparation of this book – including manuscript reading.

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# Acronyms

BG	Boltzmann–Gibbs
BGS	Boltzmann–Gibbs–Shannon
BH	Black hole
CERN	European Organization for Nuclear Research (original French name: Centre Européenne pour la Recherche Nucléaires)
FAIR	Facility for Antiproton and Ion Research
QCD	Quantum chromodynamics
EoS	Equation of state
HIC	Heavy ion collision
KMS	Kubo–Martin–Schwinger
LHC	Large Hadron Collider (at CERN)
MB	Maxwell–Boltzmann
PDF	Probability density function
RHIC	Relativistic Heavy Ion Collider (at Brookhaven, NY, US)
RN	Reissner–Nordstrøm
SPS	Superconducting Proton Synchrotron (at CERN)
SI	Système International d’Unités (International System for Measuring Units)

