Approximate Global Convergence and Adaptivity for Coefficient Inverse Problems

Larisa Beilina • Michael Victor Klibanov

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Springer

Larisa Beilina
Department of Mathematical Sciences
Chalmers University of Technology
Gothenburg University
Gothenburg, Sweden

Michael Victor Klibanov
University of North Carolina
Charlotte, North Carolina, USA

ISBN 978-1-4419-7804-2 e-ISBN 978-1-4419-7805-9
DOI 10.1007/978-1-4419-7805-9
Springer New York Dordrecht Heidelberg London

Library of Congress Control Number: 2012931927

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To our parents Yefrosiniya, Vladimir, Ada, and Victor

Preface

This book focuses on two new ideas of the authors: approximate global convergence and adaptive finite element method (FEM) for coefficient inverse problems (CIPs) for a hyperbolic partial differential equation (PDE). The first chapter might be used as an introductory course to the theory of ill-posed problems. In addition, a number of uniqueness theorems for CIPs are proved in this chapter via the method of Carleman estimates. The book features many recipes for numerical implementations of developed algorithms. Those readers who would wish to focus on numerical studies, might skip the reading of the convergence analysis. Naturally, those recipes are accompanied by many numerical examples. These examples address both synthetic (computational) and experimental data.

Two types of experimental data are studied: the data collected in a laboratory and the data collected in the field by a forward-looking radar of the US Army Research laboratory (ARL); see [126] for the description of this radar. In both cases, the most challenging case of *blind* experimental data is considered. Results of numerical testing for both synthetic and experimental data are in a good agreement with the convergence analysis. Results for ARL data address a real world problem of imaging of explosives using the data of the forward-looking radar of ARL.

Suppose that the propagation of a signal through a medium of interest is governed by a PDE. Assume that one wants to calculate a certain spatially distributed internal property of that medium using measurements of the output signal either at the entire boundary or at a part of the boundary of that medium. This property is usually described by a spatially dependent coefficient of that PDE. Thus, one arrives at a CIP for that PDE. This CIP is about the computation of that coefficient using those boundary measurements.

Having a good approximation for the coefficient of interest, one can visualize its spatial distribution. In other words, one can create an image of the interior of that medium. Hence, in simple terms, a CIP is a problem of "seeing through" the medium, i.e., this is the problem of imaging of the interior structure of that medium. Some examples of those properties of interest are spatially distributed dielectric permittivity, electric conductivity, and sound speed. It is clear from the above that CIPs have a broad range of applications in, for example, geophysics, imaging of land mines, and, more generally, hidden explosives, geophysics, and medical imaging, etc. However, having said this, the next question is: *Given measurements of an output signal, how to actually calculate the unknown coefficient of interest?*

CIPs are both nonlinear and ill-posed. These two factors combined cause very substantial difficulties in the goal of addressing this question. The very first idea which comes in mind is to minimize a least squares objective functional and approximate solution this way. However, these functionals suffer from the phenomenon of local minima and ravines. Hence, any gradient-like technique of the minimization of such a functional will likely stop at such a point of a local minimum which is the closest one to the starting point of that iteration process. Because of the local minima problem, all conventional algorithms for CIPs are locally convergent ones. This means that their convergence can be rigorously guaranteed only if the starting point of iterations is sufficiently close to the exact solution. However, a knowledge of a sufficiently small neighborhood of the exact coefficient is a luxury in the majority of applications.

Therefore, it is important for many applications to develop such numerical methods which would provide good approximations for exact solutions of CIPs without any advanced knowledge of small neighborhoods of exact solutions. This goal is an enormously challenging one. Hence, to achieve it, one can work with some approximate mathematical models. Still, these models should be verified numerically. It is also desirable to verify those approximate mathematical models on experimental data, provided of course that such data are available (usually it is both hard and expensive to get experimental data). Thus, we use a new term for such numerical methods "approximate global convergence." Results of abovementioned testing on synthetic and experimental data validate approximate mathematical models.

The development of approximately globally convergent numerical methods for CIPs with single measurement data has started from the so-called convexification algorithm [101,102,157–160], which the authors consider as an approximately globally convergent method of the first generation. The book focuses on a substantially different approach, which can be regarded as the approximately globally convergent method of the second generation. This approach was developed by the authors in 2008–2011 [9, 24–29, 109, 114–117, 160].

Only the single measurement case is considered in this book. The term "single measurement" means that only a single position of the point source or a single direction of the incident plane wave is used to generate the data. This case is preferable in, for example, military applications in which one wants to minimize the number of measurements because of many dangers on the battlefield.

The main interest in computations of applied CIPs is an accurate imaging of both locations of small inclusions as well as values of unknown coefficients inside them. Those inclusions are embedded in an otherwise slowly changing background medium. This is because those inclusions are, for example, land mines, tumors, defects in materials, etc. It is important to accurately calculate values of unknown coefficients, because they can help to identify those inclusions. We do not separate between inclusions and backgrounds. Rather we just calculate unknown coefficients. A different approach to the topic of imaging of small inclusions can be found in [4–6].

This book considers both two-dimensional and three-dimensional CIPs for an important hyperbolic PDE and addresses two questions which are the central ones for numerical treatments of those CIPs:

1. How to calculate a good approximation for the exact solution without an advanced knowledge of a small neighborhood of this solution?
2. How to refine that approximation?

The first question is addressed via a new approximately globally convergent numerical method of the authors. Corresponding approximate mathematical models basically amount to the truncation of a certain asymptotic series. The second question is addressed via a locally convergent adaptive finite element method (adaptivity). It is natural therefore that a two-stage numerical procedure is developed. On the first stage, the approximately globally convergent method provides a good approximation for the exact coefficient. On the second stage, this approximation is refined via the adaptivity. A detailed convergence analysis for both stages is an important part of this book.

The work on this book was generously supported by US Army Research Laboratory and US Army Research Office (ARO) grants W911NF-08-1-0470, W911NF-09-1-0409, and W911NF-11-1-0399, by National Institutes of Health (USA) grant 1R21NS052850-01A1, by Swedish Research Council, Swedish Foundation of Strategic Research, Gothenburg Mathematical Modeling Center and by Visby Program of Swedish Institute. We express our special gratitude to Dr. Joseph D. Myers, the Program Manager of the Numerical Analysis program of ARO.

Computations of Chaps. 3–5 and Sect. 6.8.5 were performed (1) on 16 parallel processors in NOTUR 2 production system of NTNU, Trondheim, Norway (67 IBM p575+16 way nodes, 1.9 GHz dual-core CPU, 2,464 GB memory) and (2) in a center for scientific and technical computing C3SE at Chalmers University and Gothenburg University, Gothenburg, Sweden. Computations of Chap. 6 with the only exception of Sect. 6.8.5 were performed computational facilities of the Department of Mathematics and Statistics of University of North Carolina at Charlotte, Charlotte, USA.

A number of our colleagues, who are listed below in the alphabetical order, have helped us in our work on this book. Dr. Mohammad Asadzadeh has collaborated with the first author on the development of the idea of using the adaptivity inside the approximately globally convergent method. This has resulted in the adaptive one-stage numerical procedure; see [9] and Sect. 4.17. Dr. Anatoly B. Bakushinsky has provided a significant input in our formulation of the definition of the approximate global convergence property and has also advised us many times on a number of issues of the theory of ill-posed problems; see [111] and Sect. 1.8. Dr. Christian Clason has collaborated with the first author on the subject of the application of the adaptivity technique to scanning acoustic microscopy; see [21] and Sect. 4.14.2.2. Drs. Michael A. Fiddy and John Schenk have collected experimental data in the

Microwave Laboratory of University of North Carolina at Charlotte; see Chap. 5 as well as [28, 109]. Dr. Irina Gainova has helped us with many technical issues related to the text of this book. Dr. Claes Johnson was Ph.D. advisor of the first author. He has presented to the first author the idea of the adaptivity for the CIPs, for the first time; see [16, 20] and Sects. 4.5 and 4.14. Dr. Mikhail Yu. Kokurin has collaborated with us on the topic of the accuracy improvement with mesh refinements in the adaptivity technique; see [29] as well as Sects. 1.9, 4.1.2, and 4.9. Drs. Andrey V. Kuzhuget and Natee Pantong have performed computations of the major part of results of Chap. 6; also see [114–117]. Drs. Lam Nguyen and Anders Sullivan from ARL have supplied us with experimental data collected by the forward-looking radar of ARL in the field, along with the permission to use these data in the current book; see Sect. 6.9 of Chap. 6. The corresponding joint work is [117]. Dr. Roman G. Novikov has given us a number of quite useful advises on some analytical and numerical issues. We sincerely appreciate a great help of all these individuals.

Gothenburg, Sweden — Larisa Beilina
Charlotte, North Carolina, USA — Michael Victor Klibanov

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