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Walter G. Kelley • Allan C. Peterson

The Theory of Differential Equations

Classical and Qualitative

Second Edition

Springer

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We dedicate this book to our families:
Marilyn and Joyce
and
Tina, Carla, David, and Carrie.

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Preface

Differential equations first appeared in the late seventeenth century in the work of Isaac Newton, Gottfried Wilhelm Leibniz, and the Bernoulli brothers, Jakob and Johann. They occurred as a natural consequence of the efforts of these great scientists to apply the new ideas of the calculus to certain problems in mechanics, such as the paths of motion of celestial bodies and the brachistochrone problem, which asks along which path from point P to point Q a frictionless object would descend in the least time. For over 300 years, differential equations have served as an essential tool for describing and analyzing problems in many scientific disciplines. Their importance has motivated generations of mathematicians and other scientists to develop methods of studying properties of their solutions, ranging from the early techniques of finding exact solutions in terms of elementary functions to modern methods of analytic and numerical approximation. Moreover, they have played a central role in the development of mathematics itself since questions about differential equations have spawned new areas of mathematics and advances in analysis, topology, algebra, and geometry have often offered new perspectives for differential equations.

This book provides an introduction to many of the important topics associated with ordinary differential equations. The material in the first six chapters is accessible to readers who are familiar with the basics of calculus, while some undergraduate analysis is needed for the more theoretical subjects covered in the final two chapters. The needed concepts from linear algebra are introduced with examples, as needed. Previous experience with differential equations is helpful but not required. Consequently, this book can be used either for a second course in ordinary differential equations or as an introductory course for well-prepared students.

The first chapter contains some basic concepts and solution methods that will be used throughout the book. Since the discussion is limited to first-order equations, the ideas can be presented in a geometrically simple setting. For example, dynamics for a first-order equation can be described in a one-dimensional space. Many essential topics make an appearance here: existence, uniqueness, intervals of existence, variation of parameters, equilibria, stability, phase space, and bifurcations. Since proofs of existence-uniqueness theorems tend to be quite technical, they are reserved for the last chapter.

Systems of linear equations are the major topic of the second chapter. An unusual feature is the use of the Putzer algorithm to provide a constructive method for solving linear systems with constant coefficients. The study of stability for linear systems serves as a foundation for nonlinear systems in the next chapter. The important case of linear systems with periodic coefficients (Floquet theory) is included in this chapter.

Chapter 3, on autonomous systems, is really the heart of the subject and the foundation for studying differential equations from a dynamical viewpoint. The discussion of phase plane diagrams for two-dimensional systems contains many useful geometric ideas. Stability of equilibria is investigated by both Liapunov's direct method and the method of linearization. The most important methods for studying limit cycles, the Poincare-Bendixson theorem and the Hopf bifurcation theorem, are included here. The chapter also contains a brief look at complicated behavior in three dimensions and at the use of *Mathematica* for graphing solutions of differential equations. We give proofs of many of the results to illustrate why these methods work, but the more intricate verifications have been omitted in order to keep the chapter to a reasonable length and level of difficulty.

Perturbation methods, which are among the most powerful techniques for finding approximations of solutions of differential equations, are introduced in Chapter 4. The discussion includes singular perturbation problems, an important topic that is usually not covered in undergraduate texts.

The next two chapters return to linear equations and present a rich mix of classical subjects, such as self-adjointness, disconjugacy, Green's functions, Riccati equations, and the calculus of variations.

Since many applications involve the values of a solution at different input values, boundary value problems are studied in Chapter 7. The contraction mapping theorem and continuity methods are used to examine issues of existence, uniqueness, and approximation of solutions of nonlinear boundary value problems.

The final chapter contains a thorough discussion of the theoretical ideas that provide a foundation for the subject of differential equations. Here we state and prove the classical theorems that answer the following questions about solutions of initial value problems: Under what conditions does a solution exist, is it unique, what type of domain does a solution have, and what changes occur in a solution if we vary the initial condition or the value of a parameter? This chapter is at a higher level than the first six chapters of the book.

There are many examples and exercises throughout the book. A significant number of these involve differential equations that arise in applications to physics, biology, chemistry, engineering, and other areas. To avoid lengthy digressions, we have derived these equations from basic principles only in the simplest cases.

In this new edition we have added 81 new problems in the exercises. In Chapter 1 there is a new section on the generalized logistic equation, which has important applications in population dynamics. In Chapter 2 an additional theorem concerning fundamental matrices, a corresponding example and related exercises are now included. Also results on matrix norms in Section 2.4 are supplemented by the matrix norm induced by the Euclidean norm and Lozinski's measure with examples and exercises are included. In Chapter 3 an intuitive sketch of the proof that every cycle contains an equilibrium point in it's interior has been added. Also to supplement the results concerning periodic solutions, Liénard's Theorem is included with an application to van der pol's equation. Section 3.8 has been updated to be compatible with Mathematica, version 7.0. In Chapter 5, the integrated form of the Euler–Lagrange equation has been added with an application to minimizing the surface area obtain by rotating a curve about the x-axis. Liapunov's inequality is identified and an example and exercises are included.

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