

Topological Degree Approach to Bifurcation Problems

Topological Fixed Point Theory and Its Applications

VOLUME 5

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Topological Degree Approach to Bifurcation Problems

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