

FROM *SUMMETRIA* TO SYMMETRY: THE MAKING
OF A REVOLUTIONARY SCIENTIFIC CONCEPT

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From *Summetria* to Symmetry:
The Making of a Revolutionary
Scientific Concept

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For Ruth Lorand

Preface

Many literary critics seem to think that an hypothesis about obscure and remote questions of history can be refuted by a simple demand for the production of more evidence than in fact exists. The demand is as easy to make as it is impossible to satisfy. But the true test of an hypothesis, if it cannot be shown to conflict with known truths, is the number of facts that it correlates and explains.

Francis M. Cornford [1914] 1934, 220.

It was in the autumn of 1997 that the research project leading to this publication began. One of us [GH], while a visiting fellow at the Center for Philosophy of Science (University of Pittsburgh), gave a talk entitled, “Proportions and Identity: The Aesthetic Aspect of Symmetry”. The presentation focused on a confusion surrounding the concept of symmetry: it exhibits unity, yet it is often claimed to reveal a form of beauty, namely, harmony, which requires a variety of elements. In the audience was the co-author of this book [BRG] who responded with enthusiasm, seeking to extend the discussion of this issue to historical sources in earlier periods. A preliminary search of the literature persuaded us that the history of symmetry was rich in possibilities for new insights into the making of concepts. John Roche’s brief essay (1987), in which he sketched the broad outlines of the history of this concept, was particularly helpful, and led us to conclude that the subject was worthy of monographic treatment.

The received view is that symmetry is an innate concept that was always available to human thought. There is no doubt that we moderns perceive symmetrical elements in nature as well as in artifacts produced in virtually all cultures in all periods, but is it the case that the ancients noticed what we see? Contrary to a widely held expectation, the answer is negative, for no evidence has been adduced to support the claim that the ancients were alert to this concern; rather, it is a perspective imposed by modern historians and philosophers on their forebears. Indeed, as a matter of historical fact, prior to the mid-18th century the term, *symmetry*, does not occur in any of its modern senses. Moreover, there was no term or expression to connote the meaning of the modern concept of symmetry. Typically, this unsettling negative result leads to a request for the production of more evidence than in fact exists. And, as Cornford realized, the demand is as easy to make as it is impossible to satisfy.

Despite the lack of evidence in this period for the concept, we discern two coherent trajectories of the term, *symmetry*, which together constitute a fascinating tale which has not been told heretofore. We call the first path mathematical and the second, aesthetic. Thinkers such as Plato (427–347 BC), Euclid (*fl.* 300 BC), Archimedes (287–212 BC), and Isaac Barrow (1630–1677) contributed to the formulation of the concept in the mathematical path, while Vitruvius (1st century, BC), Leon Battista Alberti (1404–1472), Claude Perrault (1613–1688), and Montesquieu (1689–1755), are the principal players in the aesthetic domain. In the mathematical path the meaning remained stable for many centuries and then fell out of active use, but in the aesthetic path we find an intriguing set of developments. These issues will be discussed in Part I.

Beginning in the mid-18th century the term, *symmetry*, was used in scientific contexts in new ways, at first in rare instances. But during the Revolutionary and Napoleonic period (1789–1815) the pace quickened, for we find definitions and applications of the term in a wide variety of scientific disciplines, notably, natural history, mathematics, and physics. Most of these usages have not been considered in the secondary literature, let alone drawn together in a connected narrative. Indeed, we have not found any discussion in the secondary literature of the usages of *symmetry* in physics in the 18th and early 19th centuries. One of the main goals of this monograph is to fill this lacuna. In particular, we discuss in detail a radical new mathematical meaning for the term, *symmetry*, complete with a precise definition, introduced by Adrien-Marie Legendre (1752–1833) in 1794. This breakthrough has not been properly appreciated, and we recount it against the failed attempt of Immanuel Kant (1724–1804) to formulate a new concept which he called “incongruent counterparts”. It is instructive to recognize that Kant, the foremost philosopher of the 18th century, struggled unsuccessfully to establish a new concept, connoting a sense akin to bilateral symmetry. Despite his failure to reach the goal he set for himself, Kant’s essay (1768) contains many brilliant insights into mathematical structures shared by objects that were otherwise considered unrelated, e.g., snails and screws. These innovations will be analyzed in Part II.

In order to justify the claim that scientific usages of *symmetry* beginning in the mid-18th century were indeed novel, we decided to investigate earlier usages of the term. In so doing, we discovered that the traditions reported in Part I inform the innovations in Part II. For example, Perrault, the translator of Vitruvius’s *De architectura* into French (1673), contrasted two meanings of symmetry, the ancient usage by Vitruvius and the French usage of his day. We suggest that this French usage in architecture, later called “respective symmetry”, was part of the background for the invocation of the term, *symmetry*, by Gaspard Monge (1746–1818) in the description of a curved figure whose center of gravity he sought to determine (1788). One should bear in mind that Perrault’s aesthetic usage expresses a value judgment, whereas Monge’s scientific usage only functions as a descriptive term in mathematical physics. Monge’s appeal to *symmetry* in a textbook on statics—addressed to students in the French naval academy—seems to be the first occurrence of the term in its modern bilateral sense in a treatise on physics, and it comes in a section dealing with the center of gravity of ships. We then realized that books on naval architecture

in the 18th century, intended for practitioners of shipbuilding, have to be considered as providing precedents for this usage. Monge did not see the need for a definition of symmetry in the bilateral sense since his usage conformed to what was already in the relevant literature of naval architecture. But it is noteworthy that, after Legendre introduced his revolutionary definition in 1794, it became increasingly common to define symmetry in various scientific domains.

Aesthetic usages play an important role in Part I but not in Part II, where our interest shifts to early scientific usages of symmetry. We seek to trace the making of the concept in modern scientific discourse and argue that it mainly took place at the time of the French Revolution and, indeed, in France. In Europe from the 16th to the mid-18th century most instances of *symmetry* occurred in the context of architecture, a discipline in which aesthetics plays a prominent role, and the usages in this tradition are essential for understanding the novelty of the modern scientific concept.

The reader, however, may ask, as indeed our colleagues have done, How do you know that symmetry in the modern scientific sense was not used before the mid-18th century? It is true that initially we ourselves believed that symmetry was present from the very beginning (i.e., Greek antiquity). Indeed, a number of passages in scientific works before the mid-18th century were suggested to us as evidence for early usages of the modern concept. But when we examined these passages, they failed to provide any supporting evidence, and this cast doubt on our initial belief and the underlying methodology. We then decided to adopt a different approach: we began to look for the way the term, *symmetry*, was actually used, rather than identifying the concept of symmetry in passages where the term does not occur. In our view, the history of a concept cannot be entirely divorced from the words used to articulate it. Such a history should take into account the variety of meanings as the application of the concept changes over time—in different contexts, different problems arise and, in turn, different answers are given. In response to these concerns we adhere to a methodology in which we avoid anachronistic readings by paying careful attention to the relevant contexts and, for us, this means primarily the text surrounding an occurrence of *symmetry* and secondarily the network of near contemporary usages. Moreover, before claiming that any idea was new in the period from the mid-18th century to 1815, we have—at the very least—checked Diderot's *Encyclopédie* (1751–1765) which serves as a reliable guide to the state of knowledge in its day.

We recognize, of course, that the scientific concept of symmetry continued to develop throughout the 19th century and beyond. Indeed, symmetry considerations have taken on an ever greater role in many disciplines, and it would probably take a team of scholars to do justice to this central aspect of science in the 19th and 20th centuries. We leave this task to our successors.

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Contents

1 Introduction	1
1.1 Setting the Problem: the Historical Context of Symmetry	1
1.2 The Perceptual Approach: Ill-Founded Expectations	5
1.3 Philosophical Perspectives	9
1.3.1 Symmetry—Special Features	24
1.4 Historiographical Considerations	27
1.5 The Argument	48

Part I Tradition: Ancient Perspectives and Their Survival in the Early Modern Era

2 The Mathematical Path	69
2.1 Introduction	69
2.2 Plato (427–347 BC) and Aristotle (384–322 BC)	70
2.3 Euclid (<i>fl.</i> 300 BC)	70
2.4 Archimedes (287–212 BC)	74
2.5 Boethius (480–524) and Robert Recorde (1510–1558)	76
2.6 Nicole Oresme (1323–1382) and the Medieval Tradition in Latin ...	79
2.7 Johannes Kepler (1571–1630) and Galileo Galilei (1564–1642) ...	82
2.7.1 Francesco Maurolico (1494–1575)	85
2.8 The Perspective of Isaac Barrow (1630–1677)	86
2.9 Isaac Newton (1642–1727)	89
2.10 The <i>Encyclopédie</i> (1751–1765) and Féraud’s Dictionary (1787–1788)	89
2.11 Conclusion	90
3 The Aesthetic Path	93
3.1 Symmetry as “Properly Proportioned”	93
3.1.1 Plato: Beauty as Symmetry	94
3.1.2 Aristotle	95
3.1.3 Geminus (1st Century, BC) and Ptolemy (<i>fl.</i> 150)	96

3.1.4	Galen (<i>ca.</i> 130–200): The Ideal Proportions of a Human Being	98
3.2	Vitruvius (1st Century, BC), <i>De architectura</i>	99
3.2.1	Symmetry in the Human Body	100
3.2.2	Symmetry in Architecture	102
3.2.3	Symmetry in Machines	105
3.3	Early Modern Times	106
3.3.1	Albrecht Dürer (1471–1528) and Joachim Camerarius (1500–1574) in Germany	106
3.3.2	Henry Wotton (1568–1639) and Isaac Barrow in England	107
3.4	Conclusion: The Richness of the Ancient Concept of Symmetry	109
4	New Aesthetic Sensibilities in Italian and French Architecture	111
4.1	The Italian Tradition: The Correspondence of Two Sides About a Central Feature	111
4.1.1	Leon Battista Alberti (1404–1472) and His Ancient Sources	112
4.1.2	Sebastiano Serlio (1475–1554)	118
4.1.3	Andrea Palladio (1508–1580)	119
4.1.4	Vincenzo Scamozzi (1552–1616)	119
4.2	The French Tradition: Towards a New Concept of Symmetry	120
4.2.1	Philibert Delorme (<i>ca.</i> 1510–1570), Louis Savot (1579–1640), Pierre Le Muet (1591–1669), and Blaise Pascal (1623–1662)	121
4.2.2	Claude Perrault (1613–1688)	127
4.2.3	Augustin Charles Daviler (1653–1700)	134
4.2.4	Entries in the <i>Encyclopédie</i> and French Dictionaries	135
4.2.5	The Metaphor of the Balance: Montesquieu (1689–1755)	140
4.2.6	Denis Diderot (1713–1784): Symmetry in Architecture and Painting	145
4.3	Diffusion of the New Concept Beyond France: William Hogarth (1697–1764) and Edmund Burke (1729–1797) in England, and Christian Wolff (1679–1754) in Germany	148
4.4	Conclusion: The Gradual Shift Away from the Ancient Meaning of Symmetry in Aesthetics	153
5	The Ancient Concept of Symmetry in Scientific Contexts in Early Modern Times and Its Association with Harmony	157
5.1	Introduction	157
5.2	Nicolaus Copernicus (1473–1543): an Architectural Conception of the Planetary System	157
5.2.1	The Impact of Vitruvius on Copernicus: a Combined Concept of Symmetry and Harmony	158
5.3	Galileo and the Old Tradition	163
5.4	Some Cases of Analysts’ Imposing the Concept of Symmetry on Early Modern Thinkers	170

5.4.1	Kepler and René Descartes (1596–1650): The Treatment of Snowflakes	170
5.4.2	Kepler and Gottfried Wilhelm von Leibniz (1646–1716): Harmony as a Fundamental Concept in Astronomy and Metaphysics	172
5.5	Conclusion	176

Part II The Path to Revolution: Symmetry as a Modern Scientific Concept

6	The Treatment of Symmetry in Natural History (1738–1815)	179
6.1	Usages of the Term, <i>Symmetry</i> , in Natural History	179
6.1.1	Botany	179
6.1.2	Crystallography	188
6.1.3	Zoology	198
6.2	Conclusion	199
7	Leonhard Euler (1707–1783) and Immanuel Kant (1724–1804): “Revolutions” That Did Not Happen	203
7.1	Introduction	203
7.2	Euler: Dual Representation of Celestial Constellations (1750)	204
7.3	Kant: Incongruent Counterparts and Symmetry (1768)	207
7.3.1	Kant and the French Architectural Tradition	211
7.3.2	Incongruent Counterparts and the Directionality of Space	212
7.3.3	Kant’s Move from Mathematics to Metaphysics	217
7.4	Conclusion	218
8	Legendre’s Revolutionary Definition of Symmetry as a Scientific Concept (1794)	221
8.1	Adrien-Marie Legendre (1752–1833): <i>Éléments de géométrie</i> (1794)	221
8.1.1	The Background to Legendre’s Work in Geometry	221
8.1.2	A New Proof for Euler’s Formula: Projecting Polyhedra onto the Surface of a Sphere	225
8.2	Robert Simson (1687–1768): Difficulties with Definitions 9 and 10 in Euclid’s <i>Elements</i> , Book XI	228
8.2.1	Simson’s Formulation of the Problem	228
8.2.2	Simson’s Critique of Commentators on Euclid: A Counter-Example	229
8.2.3	“Restoring” Euclid	231
8.3	The New Definition of Symmetry	232
8.3.1	Reexamining Euclid: Two Possible Orders of the Plane Angles that Comprise a Solid Angles	233
8.3.2	The Definition: A Relation of Similar and Equal Solid Angles Whose Plane Angles Are in Inverse Order	234
8.3.3	Symmetrical Polyhedra	237

8.3.4	The Properties of Symmetrical Polyhedra	238
8.4	Applications	242
8.4.1	Spherical Triangles	242
8.4.2	Resolving Simson’s Critique	243
8.5	Legendre vs. Kant	246
8.5.1	Mirror Image in Optics: Ptolemy, Newton, Diderot	246
8.5.2	Legendre’s Appeal to a Mirror	247
8.5.3	Comparing the Views of Legendre and Kant	249
8.6	Early Responses to Legendre’s Definition of Symmetrical Solids . . .	251
8.6.1	Sylvestre François Lacroix (1765–1843) and Jean-Guillaume Garnier (1766–1840): An “Ingenious” Concept	251
8.6.2	Meier Hirsch (1765–1851): Legendre’s Definition in a German Text (1807)	252
8.6.3	Augustin-Louis Cauchy (1789–1857): A Geometrical Proof	253
8.7	Conclusion	259
9	New Applications of Symmetry in Mathematics and Physics: 1788–1815	263
9.1	Introduction	263
9.2	Bilateral Symmetry	263
9.3	Symmetry as an Abstract Concept	281
9.3.1	The Concept of Symmetrical Events (Probability)	281
9.3.2	The Concept of a Symmetrical Function (Algebra)	286
9.4	Conclusion	291
	Coda: Legendre’s Choice of <i>Symmetry</i>—What’s in a Word?	295
	Postscript	301
	References	303
	Index	323