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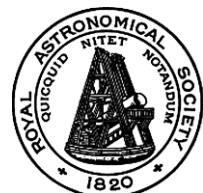
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Christian Caron
Springer Heidelberg
Physics Editorial Department I
Tiergartenstrasse 17
69121 Heidelberg / Germany
christian.caron@springer.com

Sverre J. Aarseth
Christopher A. Tout
Rosemary A. Mardling (Eds.)

The Cambridge *N*-Body Lectures

 Springer



*Advancing
Astronomy and
Geophysics*

Sverre J. Aarseth
University of Cambridge
Institute of Astronomy
Madingley Road
Cambridge CB3 0HA
United Kingdom
sverre@ast.cam.ac.uk

Christopher A. Tout
University of Cambridge
Institute of Astronomy
Madingley Road
Cambridge CB3 0HA
United Kingdom
cat@ast.cam.ac.uk

Rosemary A. Mardling
School of Mathematical Sciences
Monash University
Victoria 3800
Australia
mardling@sci.monash.edu.au

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Preface

This book gives a comprehensive introduction to the tools required for direct N -body simulations. The contributors are all active researchers who write in detail on their own special fields in which they are leading international experts. It is their previous and current connections with the Cambridge Institute of Astronomy, as staff or visitors, that gives rise to the title. The material is generally at a level suitable for a graduate student or postdoctoral worker entering the field.

The book begins with a detailed description of the codes available for N -body simulations. In a second chapter we find different mathematical formulations for special treatments of close encounters involving binaries or multiple systems, which have been implemented. The concept of chaos and stability plays a fundamental role in celestial mechanics and is highlighted here in a presentation of a new formalism for the three-body problem. The emphasis on collisional stellar dynamics enables the scope to be enlarged by including methods relevant for comparison purposes. Modern star cluster simulations include additional astrophysical effects by modelling real stars instead of point-masses. Several contributions cover the basic theory and comprehensive treatments of stellar evolution for single stars as well as binaries. Questions concerning initial conditions are also discussed in depth. Further connections with reality are established by an observational approach to data analysis of actual and simulated star clusters. Finally, important aspects of hardware requirements are described with special reference to parallel and GRAPE-type computers. The extensive chapters provide an essential framework for a variety of N -body simulations.

During an extensive summer school on astrophysical N -body simulations, held in Cambridge, www.cambody.org, the Royal Astronomical Society encouraged us to edit a volume on the topic, to be published in The Royal Astronomical Society Series. Subsequently, we collected the tutorial lecture notes assembled in this volume. We would like to take this opportunity to thank the Royal Astronomical Society for sponsoring the school and the Institute of Astronomy for provision of school facilities. We are grateful to all the authors

who took time off from their busy schedules to deliver the manuscripts, which were then checked for both style and scientific content by the editors. This collection of topics, related to the gravitational N -body problem, will prove useful to both students and researchers in years to come.

Cambridge
May 2008

Sverre J. Aarseth
Christopher A. Tout
Rosemary A. Mardling

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