THE SPECIAL THEORY OF RELATIVITY

A Title in the Nature–Macmillan Physics Series

THE SPECIAL THEORY OF RELATIVITY

H. Muirhead

University of Liverpool

Macmillan Education

© H. Muirhead 1973

Softcover reprint of the harcover 1st edition 1973 978-0-333-12844-2

All rights reserved. No part of this publication may be reproduced or transmitted, in any form or by any means, without permission.

First published 1973 by
THE MACMILLAN PRESS LTD
London and Basingstoke
Associated companies in New York Dublin
Melbourne Johannesburg and Madras

ISBN 978-0-333-12845-9

ISBN 978-1-349-86187-3 (eBook)

DOI 10.1007/978-1-349-86187-3

Preface

It has always been a source of surprise to me that no undergraduate textbooks on special relativity have been written by high-energy nuclear physicists. They are the people who most commonly use relativity, and at the same time their approach has more in common with Einstein's original work than most textbooks now show. The latter place too much emphasis (in the opinion of the present author) on what one might call the 'clocks and rods' aspect and too little on the principle of the invariance of physical laws in different reference frames.

This book is an attempt to redress the balance. I have tried to set out the exploitation of the principle of Lorentz invariance in the way in which it is done in high-energy physics, and in doing so have tried to indicate that the basic concept of the invariance of physical laws is a very general one and goes far beyond the context of relativity. Furthermore the techniques developed by high-energy physicists for the construction of physical equations by invariance arguments are an important extension of the principles of dimensional analysis and deserve to be more widely known than they are at present.

In writing a textbook on special relativity from the high-energy physicist's point of view, I have also felt the necessity to include material on spin and polarisation in relativistic situations. This is a subject of considerable importance in high-energy physics, and is readily amenable to presentation to undergraduates. Nevertheless, the topic is ignored by most writers on special relativity.

The basic plan of the book is that chapter 1 is concerned with historical matters. Chapter 2 deals with the basic Lorentz transformation, and the concept of the invariance of the interval between two events. This invariance principle, together with the implications of the rotation of space-time axes are examined in chapter 3. Chapter 4 is concerned with four-vectors and tensors, and in particular with the momentum four-vector. The treatment of spin and polarisation is made in chapter 5, and a further discussion of the invariance principle is made. Chapter 6 is concerned with dynamics, chiefly electrodynamics, and the invariance of the transformation properties of physical equations is exploited to examine the behaviour of the motion and polarisation vector ('spin') of charged particles in electrodynamic fields.

In introducing the Lorentz transformation, I was tempted to ignore the early work and worries of the turn of the century and to treat the subject axiomatically. However, I think it is good for students to know that physics rarely evolves in a neat orderly way and that a successful theory normally emerges only after a number of pathways have been tried and found to be blind alleys. For me one of the most interesting facts which emerged in writing chapter 1 was the realisation that Maxwell obviously had worries about the ether concept, and that they lead to a curiously defensive tone in his presentation of certain parts of his *Treatise on Electricity and Magnetism*.

Except in chapter 1, I have used MKS units for electromagnetic systems. They appear to me to be particularly inappropriate for a textbook on special relativity as they obscure rather than bring out the role of the velocity of light. However, as they are in general use I have reluctantly stuck to them.

I wish to thank I. G. Muirhead for reading the manuscript, and Drs J. Bailey and E. Picasso for supplying suitable diagrams for figures 6.4 and 6.5.

H. Muirhead

Contents

Acknowledgements	xi
1 Historical notes	1
1.1 Galileo and Newton	1
1.2 Revolutions in physics in the nineteenth century	5
1.3 Problems	7
1.4 The age of Einstein	11
Problems	12
2 Einstein and relativity	14
2.1 Preamble	14
2.2 The assumptions	15
2.3 The implications	16
2.4 Intervals	20
2.5 The light cone	21
2.6 Invariance properties of the interval	23
Problems	27

3 Applications of the Lorentz transformation	29
3.1 The life span of a meson	29
3.2 Clocks and rods	33
3.3 Time dilatation and Lorentz contraction	34
3.4 Simultaneity and causality	37
3.5 The clock paradox	39
3.6 The visual appearance of rapidly moving objects	42
3.7 The relativistic transformation of velocities	47
3.8 Successive Lorentz transformations	49
Problems	50
4 Four-vectors	52
4.1 Introduction	52
4.2 The energy-momentum four-vector	54
4.3 Further properties of the momentum four-vector	58
4.4 The Lorentz transformation of the momentum four-vector	60
4.5 Further examples of the Lorentz transformation of four-momenta	64
4.6 The emission of light from moving objects	69
4.6.1 The aberration of light	70
4.6.2 The Doppler shift	72
4.6.3 The intensity of the emitted light	73
4.7 The addition of four-vectors	75
4.7.1 Compton scattering	76
4.7.2 Elastic antiproton-proton scattering	78
4.8 The centre of momentum system	81
4.9 Other four-vectors, four-tensors	86
Problems	
5 Spin and special relativity	90
5.1 Angular momentum—a nonrelativistic concept?	90
5.2 The Pauli–Lubanski vector	91
5.2.1 Angular momentum in classical mechanics	92
5.2.2 Spin	93
5.2.3 The Pauli–Lubanski vector	94
5.3 The nonrelativistic limit	95
5.4 The Lorentz boost	97
5.5 The relativistic limit	99
5.6 Symmetries in physics	100
5.6.1 Symmetries and the interval	100
5.6.2 The violation of the conservation laws	101
Problems	103
6 The Lorentz invariance of physical theories	106
6.1 Maxwell’s equations and the conservation of charge	106
6.1.1 The equation of continuity	106
6.1.2 Charge conservation in Lorentz transformations	109
6.2 Maxwell’s equations and the Lorentz transformation	111

6.2.1 Reformulation of the equations	111
6.2.2 The Lorentz transformation of Maxwell's equations	113
6.2.3 The transformation of the fields $\mathbf{B}$ and $\mathbf{E}$	118
6.3 Solutions of Maxwell's equations	120
6.3.1 The effects of Galilean and Lorentz transformations upon the propagation of electromagnetic waves	120
6.3.2 Relativistic wave equations	122
6.3.3 Solutions in the Coulomb gauge	125
6.3.4 The 'spin' of a photon	127
6.4 Invariants of the electromagnetic system	131
6.5 The Lorentz force	131
6.5.1 Derivation	131
6.5.2 The magnetic deflection of charged particles	135
6.6 The precession of the polarisation of particles moving in a homogeneous electromagnetic field	137
6.6.1 Construction of the equation of motion	137
6.6.2 Solutions for the equation of motion	140
6.6.3 The ' $g - 2$ ' experiment	143
Problems	147
Appendixes	149
A.1 The Michelson–Morley experiment	149
A.2 Vector notation	152
A.2.1 The multiplication of vectors	152
A.2.2 The differentiation of vectors	154
A.2.3 Successive applications of ∇	155
A.3 Spherical triangles and the Lorentz transformation	155
Solutions to problems	158
Index	161

Acknowledgements

The author and publishers acknowledge with gratitude their indebtedness to the following sources for permission to use copuright material.

	page
The Institution of Electrical Engineers for Abstract 2277 from <i>Physics Abstracts</i> , 1905.	1
G. D. Scott and M. R. Viner for their diagram of a plane grid moving at relativistic speeds, from the <i>American Journal of Physics</i> , 334 , 534, 1964.	46
R. S. Shankland for figure 7 from <i>Atomic and Nuclear Physics</i> , Macmillan and Company Limited, 1961.	57
The Royal Society for the table of Bradley's observations of γ -Draconis, from the <i>Philosophical Transactions of the Royal Society</i> , 35 , 637, 1728.	72
A. H. Compton and the American Physical Society for the graph showing the wavelength of scattered radiation as a function of the scattering angle, from the <i>Physical Review</i> , 21 , 483, 1923.	78