NUMERICAL MATHEMATICS
EXERCISES IN COMPUTING WITH A DESK CALCULATOR

INTRODUCTORY MONOGRAPHS IN MATHEMATICS

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NUMERICAL MATHEMATICS

*Exercises in computing with a
desk calculator*

BY

A. J. MOAKES, M.A.

SENIOR MATHEMATICS MASTER, ST. PAUL'S SCHOOL

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PREFACE TO SECOND EDITION

I HAVE taken the opportunity to correct some errors and obscurities. I wish to thank correspondents and others who have made suggestions. Remaining imperfections are of course my own.

Some new material has been added, notably flow diagrams and fuller references.

A. J. MOAKES

St. Paul's School
1965

PREFACE TO FIRST EDITION

THE view is gaining ground in the educational world that to free the learner (after a certain point) from laborious calculation is to help him to grasp and develop the underlying ideas. These exercises are therefore devised to *use the machine so as to stimulate mathematical thought*. This aim has been pursued from the beginning but only becomes fully apparent after the full range of machine operations has been mastered.

This collection of exercises, along with the necessary minimum of explanation, was devised for the use of sixth-formers. It should be helpful to those teachers who want their pupils to be introduced to machines but who are not clear how to set about it.

In the first five chapters the technique of handling a machine is developed stage by stage through progressive examples. For each stage it is necessary first to learn certain manual operations, preferably by seeing them demonstrated and immediately imitating them. For a course in being, one pupil can demonstrate to another — to their mutual advantage. A teacher wishing to initiate such a course can acquire the skills with aid of (a) a full demonstration by the maker's representative, followed by (b) thorough practice, with the maker's handbook for reference.

The author wishes to express gratitude, for their encouragement and help, to Mr. M. Shoenberg (educational adviser of Olympia Business Machines, who make the Brunsviga), to Mr. F. W. Russell (education manager, Monroe Calculating Machines), to Dr. J. Crank (Head of Mathematics Department, Brunel College), to Mr. M. N. Horsman of

Brunel College, as well as to Mr. D. Grisewood of Macmillan & Co. Ltd. and to Mr. M. Bridger of Leicester College of Technology.

Acknowledgement must also be made to the author's pupils at St. Paul's School, who have greeted machines with enthusiasm, treated them with respect, and made shrewd comments on the exercises.

Standing orders for pupils doing the course are reproduced as an appendix, together with some hints which may prove useful to teachers who wish to use a machine occasionally for junior class-teaching.

A. J. M.

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Basic form of desk machine (lever-setting type)

INTRODUCTION

The Desk Calculating Machine

MACHINES operated entirely by hand invariably incorporate the following registers, where the numbers which are involved in the calculation are exhibited:

- (1) The input or *setting register*.

A number is set on the machine by the pressing of keys or the setting of levers; and the setting register indicates this number, as a check that (a) its digits have been set correctly and (b) it is correctly located.

- (2) The *cycle register* or multiplier register.

The processing of the input number is done entirely by turns of the operating handle, e.g. a single forward turn of this handle causes the input number N to be added in to the accumulator once (while still remaining set for further use). This turn is recorded as a unit figure in the cycle register. Seven such turns would show the number 7 and give $7N$ in the accumulator: this fact explains the name 'multiplier register' often employed.

- (3) The *accumulator*, already mentioned, can also be called the product register as the above example explains; but the former term explains more clearly its usual function, viz.: to accumulate (so long as no action is taken to clear it) all numbers which, singly or as multiples, are brought into it by the action of the operating handle.

The Operation of the Machine

This is described in general terms which apply to any machine except where special reference is made to a particular model.

The basic function of the *operating handle* has been described; but its operation would be very limited without the *shift lever*, a device which by lateral movement of the accumulator causes the numbers to be fed into it at any position. E.g. with the input one place to the left (relatively) the operation described in (2) would feed in not $7N$ but $70N$.

It must be possible to clear any register, and we have therefore three separate *clearing levers*, and sometimes a *total clearance lever* which clears all registers in one motion. (It is however possible, e.g. in the *Brunsviga 20*, to lock the left-hand half of the accumulator as a storage, and the clearance will then not operate on this half).

Another valuable facility on many machines is a *back-transfer device* by which a result can, while being cleared from the accumulator, also be automatically *set*. By adjustment of the shift, this setting can either be done complete or with the final digits omitted. Generally, all but the smallest machines now have this back-transfer facility. The possession of a machine with such a device eliminates one source of error (viz. resetting by hand) as well as speeding up certain programmes of calculation.

A list of firms marketing hand-machines in the U.K. is given on p. 61, with some notes on choosing a machine for teaching purposes.

Electrically driven machines are basically the same as the hand-machine in their mechanical construction, but fall into three classes according to their degree of automatic operation:

- (1) Semi-automatic machines, in which division is automatic but not multiplication;
- (2) fully automatic machines, in which both division and multiplication can be preset; and
- (3) machines like the *Marchant* in which the multiplicand is set on the keyboard and the multiplication process is then carried out digit by digit.

The possibility of jamming a machine is increased by automatic operation. It usually occurs because a new process is initiated before the machine has completed the previous one.

Note for the Learner

To the learner: the course is planned as follows:

Before each of the early chapters you must have certain machine operations shown to you on your own type of machine. These are listed in the notes at the appropriate stages. Generally, anyone who has successfully completed the exercises can act as demonstrator for you.

Repeat the manual operations until you understand them and do not let the demonstrator go until *you* are satisfied. You should then *do the exercise without help* even if it means trying it a number of times.

There are no 'answers at the back'. In every case you will be able to verify your own numerical answers by the checking method indicated.

Keep a special 'maths practical book' for this work. Record all your results, both intermediate and final; together with your method if any choice was given, and your check. A clear and systematic layout is of great importance, and in the more complex examples a specific form of layout is given for you to follow.

Where there are verbal questions, answer them in the book. Add any useful comments of your own *at the time*: this will increase the value of the record. Solution notes are given on pp. 62 onwards, to questions which seem to demand them. The existence of such a note is indicated in each case thus: [Solution note.

In professional numerical work *no* answer is given without an indication of its acceptability — by rounding off figures or otherwise. In this course you will be gradually subjecting your results to criticism; but you should from the beginning avoid copying down figures from the machine 'just because they are there'.

The book consists of 10 chapters. An exercise numbered 8.6 is the 6th exercise in the 8th chapter. The early explanatory notes precede the exercises to which they refer, but from chapter 4 onwards the left-hand pages are reserved for notes and the right-hand for the exercises.

Considerable cross-reference is used where techniques arise again in a new context, but for additional guidance you will find on each page a short summary of the contents, and at the end of the book is a short index of terms, giving in each case the page where the term first appears.