PARAMETRIC AMPLIFIERS

Philips Technical Library

Parametric Amplifiers

J. C. Decroly

L. Laurent

J. C. Lienard

G. Marechal

J. Vorobeitchik

Macmillan Education

© N.V. Philips' Gloeilampenfabrieken, Eindhoven 1973

Softcover reprint of the hardcover 1st edition 1973 978-0-333-13362-0

All rights reserved. No part of this publication may be reproduced or transmitted, in any form or by any means, without permission.

ISBN 978-1-349-81566-1 ISBN 978-1-349-81564-7 (eBook)

DOI 10.1007/978-1-349-81564-7

First published in English 1973 by

THE MACMILLAN PRESS LIMITED

London and Basingstoke

Associated companies in New York

Melbourne Dublin Johannesburg and Madras

PHILIPS

Trademarks of N.V. Philips' Gloeilampenfabrieken

To the memory of our colleague J. Vorobeitchik, whose contribution to this book was much greater than it may seem, thanks to his fruitful criticism of the whole work.

CONTENTS

Introduction

xvii

Part I: Background Noise in Parametric Amplification

1	Background noise	J. Vorobeitchik	3
1.	Background noise in telecommunications		3
1.1.	General		3
1.2.	Noise functions		7
1.3.	Narrow-band signal with high signal-to-noise ratio		12
2.	Noise factor and effective noise temperature of an amplifier		19
2.1.	Equivalent circuits of a two-port containing internal noise sources		20
2.2.	Noise factor of a two-port		25
2.3.	Effective spectral noise temperature of a passive two-port at a uniform temperature		28
2.4.	Noise factor and effective noise temperature of a chain consisting of several two-ports		30
2.5.	Effective spectral noise temperature of a frequency converter		32
2.6.	Operational temperature of a system		40
3.	Sources of background noise		42
3.1.	Internal noise sources		42
3.2.	External noise sources		44

Part II: Parametric Amplifiers for Microwave Frequencies

2	Classification of parametric amplifiers and converters	L. Laurent	53
1.	Manley and Rowe relations		53
2.	Properties derived from the Manley and Rowe relations		56

3.	Application to parametric amplification and conversion	56
4.	Three-frequency converters	57
4.1.	Sum-frequency converters	57
4.2.	Difference-frequency converters	59
5.	Four-frequency converters	61
5.1.	Sum-frequency converters	61
5.2.	Difference-frequency converters	64
6.	Negative-resistance amplifiers	64
7.	Classification of three- and four-frequency parametric systems	66
8.	Relation between the power gain and the transducer gain	66
9.	Four-frequency, double-conversion converters	67
3	Small-signal analysis	L. Laurent 70
1.	Small-signal theory for an ideal junction	70
2.	Introduction of terminal immittances in the equations	75
3.	Introduction of the varactor losses in the equations	77
4.	Introduction of reduced variables	79
5.	Special case of three-frequency systems	80
5.1.	Sum-frequency converter	80
5.2.	Difference-frequency converter	81
5.3.	Reflection amplifier	81
5.2.	Degenerate amplifiers	82
4	Three-frequency non-inverting converters	L. Laurent 83
1.	Sum-frequency converter	83
1.1.	Equivalent circuit	83
1.2.	Transducer gain	84
1.3.	Instability factor of the gain	85
1.4.	Effective noise temperature	85
1.5.	Optimization of the gain	86
1.6.	Optimization of the intrinsic noise temperature	91
1.7.	Optimization of the total noise temperature	94
1.8.	Optimization of the stability	97
1.9.	Choice of $n_{(+)}$	97
1.10.	Bandwidth	98
1.11.	Conclusions	99
2.	Difference-frequency non-inverting converters	99

5 The three-frequency inverting converter and reflection amplifier	L. Laurent	101
1. Basic equations		101
1.1. Equivalent circuit		101
1.2. Circulators and isolators		103
1.3. Transducer gain of a negative-resistance amplifier without circulator		109
1.4. Transducer gain of a negative-resistance amplifier with circulator		111
1.5. Comparison between amplifiers with and without circulator		112
1.6. Use of circulators with four and five ports		112
1.7. Use of isolators in two port amplifiers		114
1.8. The six transducer gains of difference-frequency systems		115
2. The inverting converter		121
2.1. Transducer gain		121
2.2. Instability coefficient		122
2.3. Effective noise temperature		122
2.4. Part played by the instability factor in optimizations		124
2.5. Optimization of the gain for a given instability factor		125
2.6. Optimization of the intrinsic noise temperature for a given instability factor		126
2.7. Optimization of the total noise temperature for a given instability factor		131
2.8. High-gain bandwidth		133
3. The reflection amplifier		134
3.1. Transducer gain		134
3.2. Instability factor		136
3.3. Effective noise temperature		136
3.4. Optimization of the gain for a given instability factor		138
3.5. Optimization of the noise temperature for a given gain		138
3.6. Optimization of the noise temperature for a given instability factor		139
3.7. Minimization of the total noise temperature		140
3.8. High-gain bandwidth		140
4. Conclusion		141

6	Four-frequency converters and amplifiers	L. Laurent	143
1.	Introduction		143
2.	Equivalent diagram and general equations for four-frequency converters		143
3.	Calculation of the total reduced impedances of the three branches		144
3.1.	Calculation of the total reduced input impedance $\bar{z}_e$		144
3.2.	Calculation of the equivalent circuit of the branch at $\omega_{(+)}$		145
3.3.	Calculation of the equivalent circuit of the branch at $\omega_{(-)}$		146
4.	Introduction of new reduced variables		147
5.	Calculation of the transducer gains		147
5.1.	Transducer gain of the sum-frequency converter for $f_p > f_e$		147
5.2.	Transducer gain of the sum-frequency converter for $f_p < f_e$		149
5.3.	Transducer gain of the difference-frequency converter for $f_p > f_e$		149
5.4.	Transducer gain of the difference-frequency converter for $f_p < f_e$		149
5.5.	Transducer gain of the four-frequency reflection amplifier		150
6.	Summary of the formulae when S_2 is negligible		150
7.	Stability conditions for the four-frequency converter when S_2 is negligible		151
8.	Applications		153
8.1.	The sum-frequency converter		153
8.2.	Deterioration in the performance of the reflection amplifier due to power consumption at $\omega_{(+)}$		155
7	Degenerate amplifiers	J. C. Lienard	157
1.	General study and classification		157
1.1.	Classification of difference-frequency converters and amplifiers		157
1.2.	Practical form of the three-port		160
1.3.	Calculation of the transfer functions for the input signal and noise		160
1.4.	Calculation of the transfer functions for the noise of R_s		160
1.5.	Calculation of the impedances seen by the varactor		161

1.6.	Expressions for the output signals and noise	161
1.7.	Classification of degenerate amplifiers	164
2.	Single-band reception	164
2.1.	The three methods of operation	164
2.2.	Amplifier X_e^e	165
2.3.	Amplifier $X_{e(-)}^e$	175
2.4.	Amplifier $X_{e(-)}^e$	176
2.5.	Comparison between the three amplifiers and the reflection amplifier	179
3.	Double-band reception, correlated signals	184
3.1.	The two methods of operation	184
3.2.	Amplifier $X_{e(-)}^e$	185
3.3.	Amplifier $X_{e(-)}^e$	187
3.4.	Conclusions	188
4.	Double-band reception, random signals (radiometry)	189
4.1.	The two methods of operation	189
4.2.	Amplifier $X_{e(-)}^{e(-)a}$	190
4.3.	Amplifier $X_{e(-)}^{e(-)a}$	191
4.4.	Conclusions	193
8	Pumping of parametric diodes for microwave frequencies	
	J. C. Lienard	194
1.	General	194
1.1.	Description of the pumped varactor	194
1.2.	Maximization of $ S_1 $	195
1.3.	Discussion: waveform of the pumping voltage	198
2.	Voltage pumping	198
2.1.	General formulae	198
2.2.	Calculation of the $ C_k $'s	202
2.3.	Calculation of the $ S_k $'s	205
2.4.	Calculation of the pumping power	206
2.5.	Special cases	207
3.	Current pumping	210
3.1.	General formulae	210
3.2.	Calculation of the $ C_k $'s	213
3.3.	Calculation of the $ S_k $'s	213
3.4.	Calculation of the bias	214
3.5.	Special cases	215
4.	Tables and charts for voltage pumping	218
4.1.	Introduction	218
4.2.	Mean value of the elastance	219
4.3.	First harmonic of the elastance	222
4.4.	Higher harmonics of the elastance	224

4.5.	Polarization	225
4.6.	Pumping power	226
5.	Tables and charts for current pumping	228
5.1.	Introduction	228
5.2.	Mean value of the elastance	228
5.3.	First harmonic of the elastance	231
5.4.	Higher harmonics of the elastance	233
5.5.	Polarization	233
5.6.	Pumping power	236
6.	Pumping optimization	237
6.1.	General. Pumping of negative-resistance amplifiers and sum-frequency converters	237
6.2.	Square-wave pumping	238
6.3.	Comparison between voltage and current pumping	239
6.4.	Comparison of varactors	242
9	Measurement of parametric amplifiers	J. C. Lienard 246
1.	The circle diagram and its interpretation	247
1.1.	Properties of the Smith chart	247
1.2.	Analysis of the pump circle diagram	249
1.3.	Circle diagrams of small-signal circuits	252
1.4.	Sum-frequency converter adjusted for maximum gain	255
1.5.	Sum-frequency converter adjusted for minimum intrinsic noise	256
1.6.	The inverting converter adjusted for minimum intrinsic noise	257
1.7.	The reflection amplifier adjusted for minimum noise	258
2.	Adjustment of a parametric amplifier	258
2.1.	Adjustment of the pump circuit	258
2.2.	Adjustment of a reflection amplifier	260
3.	Measurement of varactors	261
3.1.	Equivalent circuit of a varactor	261
3.2.	Characteristics specified by the manufacturer	263
3.3.	Measurement of the total capacitance	264
3.4.	Measurement of the quality factor Q	264
3.5.	Measurement of L_c	266
3.6.	Measurement of C_c and of the non-linear capacitance	266
4.	Equivalent circuits of passive two-ports	267
4.1.	Representation of the circuit elements	267

4.2.	Chain matrix of elementary two-ports	268
4.3.	Discussion: practical importance of the various equivalent circuits	269
4.4.	Study of the usual equivalent circuits	269
4.5.	Measurement of equivalent circuits by continuous displacement of a short circuit	269
10	Measurement of the noise temperature of an amplifier	
	L. Laurent	276
1.	Equipment common to all methods of measurement	276
2.	Direct measurement of the noise temperature	277
3.	Measurement by comparison with a 'monochromatic' signal generator	278
4.	Hot and cold resistance method	278
5.	Use of a thermionic diode	281
6.	Use of a gas-discharge tube	285
7.	Automatic measurement of noise temperature	288
8.	Use of a directional coupler and a cooled termination	291
11	Technical aspects	
	L. Laurent	297
1.	Reactive networks including a varactor	297
2.	Construction of reactive networks corresponding to the different branches	299
3.	Filters	301
3.1.	Types of filters most frequently used	301
3.2.	Measurement of response curves and short-circuit planes	302
4.	Location of impedance transformers	303
5.	Description of a sum-frequency converter	303
6.	Description of a reflection amplifier	306
6.1.	The parametric amplifier	306
6.2.	Auxiliary circuits of the reflection amplifier	307

Part III: Low-Frequency Parametric Amplifiers

12	Parametric amplification of low-frequency signals	
	J. C. Decroly	311
1.	Introduction	311
2.	General equations	314
3.	Analysis of the operation for $\omega_e = 0$	319
3.1.	Phase and amplitude of the output signal of the parametric amplifier	320

3.2.	Admittance Y_{in_0}	321
3.3.	Discussion of the relations (12.29) and (12.30)	321
3.4.	Operation of the parametric amplifier when $R_e Y_{\text{in}_0} \ll 1$	322
3.5.	Synchronous detection of the output signal	324
4.	Analysis of the operation when $\omega_e \neq 0$	326
4.1.	Phase and amplitude of the components of frequency f_+ and f_- of the voltage at the output of the parametric amplifier	327
4.2.	Instantaneous voltage energizing the detector	329
4.3.	Synchronous detection of the output signal	329
4.4.	Bandwidth at 3 dB	331
4.5.	Input admittance	332
5.	Background noise	334
5.1.	Internal noise sources in parametric amplifiers	334
5.2.	Noise voltage at the input of the detector	334
5.3.	Spectral density n_d of the parasitic signal at the output of the detector	337
5.4.	Noise factor	339
6.	Special case in which $\alpha = 0$; tuned-load modulator	340
13	Decoupling and methods of adjustment	G. Marechal 342
1.	Balancing of the bridge	342
1.1.	Preliminary conditions for satisfactory balancing	342
1.2.	Conditions for equilibrium	352
1.3.	Sensitivity of the trimming components	358
1.4.	Method of adjustment	364
1.5.	Special type of amplifier for d.c. signals	365
2.	Auxiliary circuits	366
2.1.	Overall block diagram	366
2.2.	Conventional amplifier	367
2.3.	Pump oscillator	368
2.4.	Demodulator	368
2.5.	Internal correction networks	368
2.6.	Automatic gain control	369
2.7.	Anti-saturation circuit	369
3.	Technical limitations	369
4.	Examples	379
5.	Details of amplification of d.c. and low-frequency signals by the parametric varactor method	381
5.1.	Input impedance	381
5.2.	Noise	382
5.3.	Gain	382

5.4.	Transformation of d.c. or low-frequency signals into a.c. high-frequency signals	382
5.5.	Bandwidth	383
5.6.	Remarks	383
14	The pumping of parametric diodes for low frequencies	
	G. Marechal	387
1.	General	387
2.	Cissoidal voltage pumping	390
2.1.	Calculation of the C'_k 's	390
2.2.	Calculation of the G'_k 's	403
2.3.	Calculation of the I'_k 's	405
3.	Rectangular voltage pumping	406
3.1.	Calculation of the C'_k 's	406
3.2.	Calculation of the G'_k 's	412
3.3.	Calculation of the I'_k 's	412
3.4.	Recurrence formulae of C_{2k+1} , G_{2k+1} , I_{2k+1} ($k \geq 2$)	412
4.	Specially important case: $I_0 = 0$	413
4.1.	Relation between V_0 and $2V_p$ for $I_0 = 0$	413
4.2.	Value of G_0 for $I_0 = 0$	413
4.3.	Influence of temperature	414
4.4.	Nomographs	415
4.5.	Practical example	416
5.	Reduced variables and important ratios	418
6.	Comparison between cissoidal and rectangular pumping	422
6.1.	Comparison of the C_1/C_0 's with β_v given	423
6.2.	Comparison for equal losses	423
6.3.	Comparison of losses for given V_0 and $2V_p$	426
6.4.	Conclusion	426
15	Measurement of the parameters of the varactor	429
	G. Marechal	
1.	Measurement of I_s , a , G_s , R_s	430
1.1.	Measurement of the static characteristic	430
1.2.	Determination of a and I_s	431
1.3.	Determination of G_s	432
1.4.	Determination of R_s	433
2.	Measurement of C_{ov} , ϕ , n , C_c	433
2.1.	Measurement of $C = C(v)$ and of $dC(v)/dv$	433
2.2.	Determination of C_{ov} , ϕ , n , C_c	434
3.	Cut-off frequency—quality factor	436
	Index	439

INTRODUCTION

It has been known since the last century that a mechanical system or an electrical circuit in which there is a parameter which varies periodically with time, may oscillate under certain conditions, but it is only quite recently that it has been suggested that use should be made of this periodic variation for the design of amplifiers. Landon described an amplifier of this type in 1949. Several years later, Buhl suggested the use of two circuits coupled by a variable gyromagnetic effect, for a microwave-frequency amplifier. But real progress in the field of 'parametric amplifiers' dates from the development by Uhler of new semiconductor diodes in which the barrier capacitance can be modulated at microwave frequencies by acting on the biasing (inverse) of the diodes. Since then, there has been a continual increase in the amount of theoretical and experimental work devoted to parametric amplifiers, providing evidence of great interest shown in this type of amplifier for both *microwave frequencies* and *low frequencies*.

The chief advantage of parametric amplifiers lies in the possibility of obtaining a low level of background noise. This feature is of particular interest in the case of transmissions in which the level of the noise, due to external sources, is very low (e.g. space telecommunications) and where, consequently, the sensitivity of the receiver depends chiefly on its intrinsic noise sources. In the case of parametric amplifiers using semiconductor diodes, with which this work is concerned, there is a further advantage, namely the possibility of having a very high input impedance at low frequencies, which opens a wide field of application for instrumentation to this type of amplifier.

Why does recourse to parametric amplification allow one to obtain noise levels lower than with the usual type of amplifier? In order to answer this question, let us remember first that an amplifier consists of both passive elements and active energy sources, the whole being terminated by two ports, one receiving the signal to be amplified and the other delivering the amplified signal. In the ordinary type of amplifier, the energy sources consist of d.c. sources which are incorporated into a network of passive elements which may be linear or non-linear. Now, it can be shown† that the essential condition

† R. J. Duffin, 'Impossible behaviour of nonlinear networks', *Journal of Applied Physics*, vol. 26, pp. 603–605, May 1955.

for such a system to act efficiently as an amplifier is; that at least one of the passive elements of the network should consist of a *resistance in which the current-voltage characteristic should not be of the quasi-linear type*, as defined by the following conditions.

- (a) The relation between the current i through the resistance and the applied voltage v is biunivocal and is independent of any other voltage.
- (b) The differential resistance dv/di is always positive.

It will thus be seen that ordinary amplifiers necessarily include a non-quasilinear element. Examples are: a triode or a transistor (which do not satisfy condition (a), since the relation $i = f(v)$ depends on the voltage of the control electrode), or a tunnel diode (which does not satisfy condition (b)).

Unfortunately, in spite of the improvements made in these devices, their operation is accompanied by background noise which is an inevitable result of their dissipative character. In order to obtain an amplifier with a low noise level, the energy must no longer be supplied to the network direct from d.c. sources, and so other methods of transfer must be envisaged. One of these is to employ a.c. sources. It can then be shown† that if a *non-linear reactance* is incorporated in the network, a system of this kind may be used efficiently as an amplifier. This method is much more interesting than the previous one, since a non-linear reactance does not generate any noise in the ideal state (in which it would be free from loss). Now, parametric diodes or varactors which are produced, in practice, behave as almost ideal non-linear reactances, both at microwave and at low frequencies. On the other hand, since the susceptance of varactors is very low at low frequencies, their use in parametric amplifiers makes it possible to obtain a very high input impedance at low frequencies. It is thus easy to appreciate the increasing importance which parametric amplifiers of the varactor type, are assuming in fields as different as those involving microwaves and very low frequencies.

This book is divided into three parts.

Part I

The first part (Chapter 1) deals with background noise. First of all the problem of background noise in telecommunications is discussed.

† J. M. Manley and H. E. Rowe, 'Some general properties of nonlinear elements; Part I, General energy relations', *Proc. I.R.E.*, vol. 44, pp. 904-913, July 1956.

This is followed by generalized definitions of the noise factor and the effective and operational noise temperatures. Next general expressions are derived for the parameters of the different types of parametric amplifiers and converters studied in this book. The chapter ends with a short discussion on the properties of the chief sources of noise, both internal and external.

Part II

In the second part we discuss parametric amplifiers for microwave frequencies and the Manley and Rowe relations are dealt with in Chapter 2. These relations are taken as the basis for the classification of parametric amplifiers and converters, and this classification is shown in a table at the end of the chapter. In Chapter 3 we introduce the small-signal hypothesis which enables the basic equations of a parametric system to be linearized. Chapter 4 deals with the calculation of the transducer gain of the effective noise temperature, and of the instability coefficient of non-inverting converters. After the general formulae have been established and discussed, the optimum operating conditions are determined, the results obtained being set out in the numerical tables. Chapter 5 deals with inverting converters and reflection amplifiers. After having given the basic equations common to these two types of amplifier, and having reviewed briefly the properties of non-reciprocal elements, we pass on to the determination of the gain, the noise temperature, the instability coefficient, the bandwidth and the optimum operating conditions. In Chapter 6 we give the general expressions for the four-frequency amplifier gains. Chapter 7 concerns degenerate amplifiers. First we establish a classification for this type of amplifier, and then consider in detail the gain, the effective noise temperature, the instability coefficient and the bandwidth. Chapter 8 is devoted to the theory of the pumping of parametric diodes, dealing with voltage and current pumping. Numerous tables and curves enable the results to be used immediately. Chapter 9 treats of the measurements required for the calculation, adjustment, and testing of parametric amplifiers, except for noise measurement which is dealt with in Chapter 10. Finally, Chapter 11 discusses some technological features peculiar to parametric amplifiers for microwave frequencies, and illustrates them by means of practical examples.

Part III

The third part deals with low-frequency parametric amplifiers. In Chapter 12 are given the general equations governing the operation of a parametric amplifier for low-frequency signals. Particular emphasis is laid on the necessity of giving this type of amplifier a differential structure by the use of two semiconductor diodes. Chapter 13 deals with the manufacture and adjustment of these amplifiers. First a study is made of the conditions for equilibrium of the differential structure introduced in Chapter 12, and a method of achieving this is given. Next there is an example of optimization, taking into account the technological limitations. Chapter 14 deals with the pumping of low-frequency diodes. Here again, the results obtained are given in the form of tables and curves. Finally, Chapter 15 describes the methods of measurement of these diodes.

This book is based partly on work done at M.B.L.E. under the auspices of the *Institut pour l'Encouragement de la Recherche Scientifique dans l'Industrie et l'Agriculture* (I.R.S.I.A.). The authors wish to thank the *Institut* for their permission to publish this section of the study. They are grateful also to their colleagues in the *Laboratoire de Recherches de la M.B.L.E.* for the computation of the tables included in this book.

J. C. DECROLY*

L. LAURENT**

J. C. LIENARD**

G. MARECHAL**

J. VOROBETCHIK***

* Formerly with M.B.L.E., now with Disment Boort, Brussels.

** M.B.L.E., Brussels.

*** Deceased, formerly with M.B.L.E.