Counting and Counters

By the same author

Digital Circuits for Binary Arithmetic

Counting and Counters

R. M. M. Oberman

*Professor in Information Engineering
Technical University
Delft, The Netherlands*

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Counting starts with zero and not with one

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Preface

During the forty years in which the design of switching circuits has been increasingly developed as a science, the design of counters has, in my experience, always been an excellent proving ground for anyone who has mastered Boolean algebra for the design of gate circuits and has acquired some knowledge of the design of sequential circuits that perform shifting and counting operations.

Counters are sequential circuits with a well-defined basic operation. They can be designed through an operating algorithm or a system description; equally, they can be designed using operating tables and Karnaugh maps, from which the final switching equations are derived.

Both methods are demonstrated in the text. However, the design method using algorithms or system concepts has been given preference because in the author's opinion this method provides the designer with the best insight into what he is really doing.

This book is intended to be used at the graduate level of study in digital electronics. Counting has always been and will always be an important operation in any automated technical system or organisation. It is therefore an important tool for the education of students in the science of digital techniques. A knowledge of counters is not only important in itself, it is even more important from the point of view of the insight it provides into the solution of switching problems.

A number of the counter designs discussed in the text have been used as student projects which could be carried out instead of a formal examination. The results of this type of student assessment have been extremely good.

The foregoing remarks about the scope of this book might give the impression that it is intended for teaching purposes only, but this is not so. The book contains more material than should be taught in a single course. Many different types of counting problem, a number of which are published for the first time, have been treated; thus the text will also be useful for many people working on the design of switching circuits. They will need this book as a reference and an explanation of the operation of commercially available counter circuits in integrated form. There are many different types of counter since counting is a part of almost every type of switching circuit.

In this book the concept of counting is treated very broadly. The author considers any circuit running through a cycle of states to be a counter. Its output may have a clear and well-defined relationship with the number of input pulses, but this relationship can also have a pseudo-random character, equally well-defined, but very difficult to state in the form of mathematical equations.

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R. M. M. OBERMAN