DECISION-MAKING UNDER UNCERTAINTY

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Tapan Biswas

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To Sabita and Raja

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Preface

This book has emerged as result of teaching economics of uncertainty and game theory for several years. To some extent, all of us differ in our perception of the subject and the relative importance of the topics included in it. Naturally, it reflects our research interests as well as the school of thought which moulded our academic upbringing. There are several books written on this subject. Each differs from others in its style and emphasis in addressing relevant issues. Each book discusses some issues informally and employs relatively rigorous methods of analysis for some other issues. This book is not an exception to this. However, there are some important features of the book which should be underlined. This book contains a chapter on non-expected utility theory, which has received very little attention in books of similar nature. In recent years, there has been an explosion of research papers in this area. It is high time that this is recognized as a core area of microeconomic research. In a book of this nature, the scope of discussion is limited. I only hope that some graduate students and their teachers will find this chapter useful. The chapters on risk aversion, stochastic dominance and the mean-variance utility theory discuss more up-to-date materials than are normally found in books on advanced microeconomics and decision theory. The theory of decision-making under uncertainty plays a significant role in issues related to justice and freedom. The last chapter in Part I (Chapter 8) contains a brief discussion on these issues, which I think is an interesting area for further economic research.

The topics covered in Part I of the book (economics of uncertainty) are by no means exhaustive. Like every other author, I had to make a painful decision on the materials not to incorporate in the book. However, I tried to make sure that a consistent development of the ideas was not compromised by such decisions. I am also convinced that too many topics and examples confuse the reader and blur the focus of the book.

Part II of the book (games and decisions) discusses and illustrates the use of game theory in the context of decisions under uncertainty. This part should not be used as a substitute for reading books on game theory and there are some excellent books on the subject. However, it would help in designing a short course aimed at introducing various concepts of game theory to the students of economics and other social sciences.

The level of mathematical sophistication varies from chapter to chapter. Chapters 1, 3, 6 and 8 in Part I and most of Part II presuppose a very basic knowledge in mathematics. Some parts of other chapters require knowledge in some advanced areas of mathematics. I have tried to water down the difficulties as far as possible. Hopefully, a reader with a little more than basic knowledge in mathematics will easily be able to cope with them.

Although the book may be used as a textbook for advanced courses, it also contains ideas in certain sections of the book which needs to be explored in future research developments. The book is not merely a survey of the literature but a critical analysis of the various developments in the subject. However, the vast scope of our subject and limitations of space and time restricted a more probing analysis of some of the topics discussed. This is a statement of fact and not an excuse; everybody writing a book of similar nature must face the same problem. This book owes a lot to Stephen Rutt (Macmillan Press) for his continuous support. A sabbatical leave from the University of Hull at the right time proved crucial. I am also grateful to my wife Sabita for her support during the course of writing this book.

It is an irony, that I ended up writing a book on economics of uncertainty. The transition of research interest from the certain environment of general equilibrium theory to decision-making under uncertainty took a long time. But it has been useful. Undoubtedly, I owe my greatest intellectual debt to Professor Lionel W. McKenzie, of the University of Rochester, whose academic commitment has been a guiding light in the career of his many students who are at the forefront of economic research today. There are many others, teachers and colleagues, whose inspirations and advices provided the backbone of this book. I am grateful to all of them.

TAPAN BISWAS