Essentials of electromagnetism

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MACMILLAN PHYSICAL SCIENCE SERIES

Essentials of electromagnetism

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List of physical constants and conversion factors

Physical constants

<i>Quantity</i>	<i>Symbol</i>	<i>Value</i>
electric constant (permittivity of free space)	ϵ_0	$8.854 \times 10^{-12} \text{ F m}^{-1}$
magnetic constant (permeability of free space)	μ_0	$4\pi \times 10^{-7} \text{ H m}^{-1}$
speed of light in a vacuum	$c = (\epsilon_0 \mu_0)^{-1/2}$	$2.998 \times 10^8 \text{ m s}^{-1}$
electronic charge	e	$1.602 \times 10^{-19} \text{ C}$
electron rest mass	m_e	$9.109 \times 10^{-31} \text{ kg}$
proton rest mass	m_p	$1.673 \times 10^{-27} \text{ kg}$
Planck's constant	h	$6.626 \times 10^{-34} \text{ J s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34} \text{ J s}$
Bohr magneton	$\mu_B = e\hbar/2m_e$	$9.274 \times 10^{-24} \text{ A m}^2$
Boltzmann's constant	k	$1.381 \times 10^{-23} \text{ J K}^{-1}$
Avogadro's number	N_A	$6.022 \times 10^{23} \text{ mol}^{-1}$

Conversion factors

1 atomic mass unit (u)	$= 1.661 \times 10^{-27} \text{ kg}$
1 electron volt (eV)	$= 1.602 \times 10^{-19} \text{ J}$
1 Angstrom (Å)	$= 10^{-10} \text{ m}$

Preface

This book presents an introduction to electromagnetism in a way that differs from that followed by most undergraduate texts. Usually such texts choose to follow a rather historically determined course. They advance along a broad front, slowly developing matters of fundamental principle (Maxwell's *in vacuo* field equations) side by side with technical applications and more phenomenological accounts of electromagnetism (Maxwell's equations in material media). Excellent though many of these texts are, the author believes that there is need of an alternative approach; one that presents a modern view of the unity of electromagnetism in as direct a way as possible. This book attempts to meet this need. It is based on lectures given in the joint honours physics course at Keele University.

The guiding principle here is the premise that 'electromagnetism is Maxwell's equations' in much the same way as, for example, 'Newton's laws are classical mechanics'. Just as a course on classical mechanics would be based at the outset on Newton's laws so here Maxwell's equations are introduced at the earliest opportunity, in order to serve as the basis for everything that follows. Electrostatics and Magnetostatics are treated as special cases of Maxwell's equations rather than as stepping stones leading to them. Similarly, the empirical laws on which the subject is founded, the laws of Coulomb, Ampère, Faraday, etc., are viewed as deductive consequences of Maxwell's equations rather than the other way round.

By forsaking the traditional historically ordered development for a more logically ordered one, the unity of electromagnetism can be made clear at an early stage. Since Maxwell's equations *in vacuo* are presented first, their physical significance can be fully discussed before introducing the phenomenological versions of these equations that apply to material media, thus allowing clearer distinctions to be made between microscopic and macroscopic fields. Despite these very worthwhile advantages it must be admitted that this approach involves a greater use of mathematical language at an earlier stage and this will be seen by some to be a significant disadvantage. Whether it is better to present all the

mathematical language of field theory (vector analysis) at the outset, as the present approach demands, or to develop it more slowly by distributing its presentation throughout the text, as the more traditional approach allows, is a matter of personal judgement. In the end the mathematics needed is the same whichever approach is followed and so for many the balance of advantage will favour the first option and the more intellectually stimulating approach to electromagnetism that it allows.

The numerical order of the chapters represents a more or less natural development of the subject, determined either by the fundamental significance of the material or by its practical importance. However, as the following flow diagram of logical connections shows, different orderings are possible. Circuit theory in chapters 9 and 10 could for example be treated after chapter 5, or, as the broken line indicates, even earlier if the concepts of capacitance and inductance are dealt with first. For those students with sufficient prior knowledge of vector analysis, chapter 2 is not essential but still useful for reference purposes.

FLOW DIAGRAM

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