Introductory Mathematics for Economics and Business

Introductory Mathematics for Economics and Business

Second Edition

K. Holden and A. W. Pearson

M

© K. Holden and A. W. Pearson 1975, 1983, 1992

Softcover reprint of the hardcover 2nd edition 1992 978-0-333-57649-6

All rights reserved. No reproduction, copy or transmission of this publication may be made without written permission.

No paragraph of this publication may be reproduced, copied or transmitted save with written permission or in accordance with the provisions of the Copyright, Designs and Patents Act 1988, or under the terms of any licence permitting limited copying issued by the Copyright Licensing Agency, 90 Tottenham Court Road, London W1P 9HE.

Any person who does any unauthorised act in relation to this publication may be liable to criminal prosecution and civil claims for damages.

First published 1975 by
David & Charles (Holdings) Ltd

Published as *Introductory Mathematics for Economists*
1983 by The Macmillan Press Ltd
Reprinted 1986, 1988, 1990

This edition published 1992 by
THE MACMILLAN PRESS LTD
Houndmills, Basingstoke, Hampshire RG21 2XS
and London
Companies and representatives
throughout the world

ISBN 978-0-333-57650-2 ISBN 978-1-349-22357-2 (eBook)
DOI 10.1007/978-1-349-22357-2

A catalogue record for this book is available
from the British Library

Contents

<i>Preface</i>	xi
1 Linear Equations	1
1.1 Some preliminaries	1
1.2 Graphical representation	4
1.3 Intercept and slope	6
1.4 Interpolation and extrapolation	7
1.5 Exercises	8
1.6 Simultaneous linear equations	9
1.7 Demand and supply	13
1.8 Exercises	16
1.9 The effects of taxation on demand	17
1.10 Simple national income models	21
1.11 IS–LM analysis	24
1.12 Exercises	26
1.13 Determinant notation	28
1.14 Exercises	35
1.15 Applications of determinants	36
1.16 Exercises	39
1.17 Revision exercises for Chapter 1	40
2 Elementary Matrix Algebra	43
2.1 The matrix notation	43
2.2 Elementary matrix operations	44
2.3 Exercises	49
2.4 The inverse matrix	50
2.5 Exercises	57
2.6 Inversion by Gaussian elimination	57
2.7 Exercises	63
2.8 Linear dependence and rank	64
2.9 Exercises	68
2.10 Further properties of matrices	69
2.11 Exercises	75

2.12	Input–output analysis	76
2.13	The input–output matrix	81
2.14	Exercises	88
2.15	Revision exercises for Chapter 2	90
3	Non-linear Equations	92
3.1	The quadratic	92
3.2	Exercises	97
3.3	The roots of a quadratic equation	98
3.4	Exercises	105
3.5	Other non-linear functions	105
3.6	Exercises	113
3.7	Breakeven point	115
3.8	Simultaneous quadratic equations	119
3.9	Net revenue	120
3.10	Exercises	124
3.11	Discontinuous functions	125
3.12	Exercises	127
3.13	Revision exercises for Chapter 3	128
4	Series	130
4.1	Introduction	130
4.2	Arithmetic progressions	130
4.3	Exercises	133
4.4	Geometric progressions	134
4.5	Exercises	136
4.6	Discounting	137
4.7	Exercises	140
4.8	Annuities and sinking funds	140
4.9	Exercises	145
4.10	Interest paid continuously	146
4.11	The binomial theorem	148
4.12	The exponential series	151
4.13	Exercises	155
4.14	Logarithms	156
4.15	Exercises	161
4.16	Revision exercises for Chapter 4	162
5	Differential Calculus	165
5.1	Introduction	165

5.2	Some general rules	169
5.3	Exercises	176
5.4	Elasticity of demand	177
5.5	Marginal analysis	180
5.6	Exercises	183
5.7	Maxima and minima	184
5.8	Cost and revenue analysis	194
5.9	Exercises	197
5.10	Profit maximisation in several markets	198
5.11	Maximising tax revenue	200
5.12	Inventory models	202
5.13	Exercises	206
5.14	Differentials	207
5.15	Taylor's and MacLaurin's theorems	209
5.16	Euler relations	212
5.17	Newton's method	213
5.18	Plotting graphs	217
5.19	Exercises	220
5.20	Revision exercises for Chapter 5	221
6	Integral Calculus	224
6.1	Introduction	224
6.2	Exercises	225
6.3	Techniques of integration	226
6.4	Exercises	228
6.5	The calculation of areas	229
6.6	Exercises	233
6.7	Absolute area	234
6.8	Exercises	235
6.9	Consumer's surplus and producer's surplus	236
6.10	Areas between curves	238
6.11	Exercises	242
6.12	Numerical methods of integration	242
6.13	Exercises	247
6.14	More difficult integration	248
6.15	Exercises	254
6.16	Revision exercises for Chapter 6	254
7	Partial Differentiation	257
7.1	Functions of more than one variable	257

7.2	Marginal analysis	263
7.3	Elasticity of demand	264
7.4	Exercises	266
7.5	Differentials	267
7.6	Total derivatives	273
7.7	Exercises	274
7.8	Implicit differentiation	275
7.9	Exercises	277
7.10	Homogeneous functions	278
7.11	Euler's theorem	280
7.12	Exercises	282
7.13	Maxima and minima	282
7.14	Exercises	285
7.15	Maxima and minima for functions of more than two variables	286
7.16	Constrained maxima and minima	289
7.17	A linear expenditure system	292
7.18	Exercises	295
7.19	Revision exercises for Chapter 7	295
8	Linear Programming	297
8.1	Introduction	297
8.2	A product-mix problem	297
8.3	A graphical approach	299
8.4	The simplex method	301
8.5	A summary of the simplex method	308
8.6	Incremental values	310
8.7	The general problem	312
8.8	Exercises	315
8.9	Revision exercises for Chapter 8	317
9	Differential Equations	319
9.1	Introduction	319
9.2	First-order linear differential equations	321
9.3	Applications of first-order differential equations	332
9.4	Exercises	336
9.5	Second-order homogeneous differential equations	338
9.6	Exercises	345
9.7	Second-order non-homogeneous differential equations	345
9.8	Exercises	349

9.9	Graphical presentation	350
9.10	Exercises	354
9.11	An inflation–unemployment model	354
9.12	Exercises	360
9.13	Revision exercises for Chapter 9	361
10	Difference Equations	364
10.1	Introduction	364
10.2	Compound interest and the addition of capital at yearly intervals	365
10.3	First-order linear difference equations	367
10.4	The cobweb model	368
10.5	The Harrod–Domar growth model	374
10.6	A consumption model	375
10.7	Exercises	376
10.8	Samuelson’s multiplier–accelerator model	377
10.9	Second-order linear difference equations	378
10.10	A consumption–investment model	384
10.11	An inflation–unemployment model	387
10.12	Exercises	391
10.13	Revision exercises for Chapter 10	393
<i>Appendix A</i>	<i>Trigonometric functions</i>	395
	<i>A.1 Definitions</i>	395
	<i>A.2 Compound angles</i>	396
	<i>A.3 Degrees and radians</i>	398
	<i>A.4 General angles</i>	399
	<i>A.5 Differentiation</i>	403
	<i>A.6 Integration</i>	404
	<i>A.7 Inverse functions</i>	406
<i>Appendix B</i>	<i>Set Theory</i>	408
	<i>B.1 Introduction</i>	408
	<i>B.2 Combining sets</i>	409
	<i>B.3 Venn diagrams</i>	410
	<i>B.4 Relations and functions</i>	414
<i>Appendix C</i>	<i>Answers to Exercises</i>	417
	<i>Further Reading</i>	506
	<i>Index</i>	509

Preface

This book introduces to students with a limited mathematics background the essential mathematics needed for economics and business courses. It is not intended to replace more formal mathematics texts and, by design, does not include proofs and derivations of all the formulae used; these are included only where they aid understanding. We hope that this approach will be suitable for those students who, as a result of their earlier experiences in this subject area, do not regard themselves as having any mathematical ability. While reading the book they are advised to check the algebra by jotting the working down on paper and to use a calculator to check the numbers. Many examples are included in the text and the exercises (with worked answers given in Appendix C) are intended to help such students. At the end of each chapter there are also some revision exercises (without answers).

The material is selected so as to increase in difficulty as the book progresses. Chapter 1 revises many basic concepts in algebra and introduces linear equations, with immediate applications in simple economic models of markets and the national economy. Chapters 2 and 3 are the natural generalisations of elementary matrix algebra and non-linear equations. Chapter 4, on series, covers many applications in finance as well as providing the groundwork for calculus in Chapters 5–7. Chapter 5, on differential calculus, includes profit maximisation for a firm, simple inventory models and other applications of marginal concepts. Integration, the subject of Chapter 6, covering both standard analytical techniques and numerical methods, is also applied to simple marginal concepts, including consumer's and producer's surplus. Relationships with many variables are examined in Chapter 7, on partial differentiation, which ends with maximisation subject to a constraint. A variation on this problem is treated in Chapter 8, on linear programming. Chapters 9–10 consider dynamic relationships – in continuous terms in Chapter 9 and in discrete terms in Chapter 10. There is an extensive treatment of trigonometric functions in Appendix A, and an introduction to set theory in Appendix B. Each of these can be treated as optional, with little loss of the main text, except for parts of Chapters 9 and 10 which

need trigonometric functions. Detailed numerical answers to the exercises are provided in Appendix C.

The order in which the material is covered can be varied. For example, integral calculus may be followed by differential equations, or matrix algebra may be deferred to precede linear programming.

We have intentionally excluded any coverage of statistics, since we believe that any brief treatment would be inadequate for most courses. We have also ignored computer programming because the wide variety of languages and packages now available mean that any general discussion is unhelpful to students and teachers. The omission of these topics should not be taken as evidence that we believe they are unimportant.

We are grateful to Roger Latham (Queen's University, Ontario, Canada) and David Peel (University College, Aberystwyth), formerly of the University of Liverpool, for reading through an early draft of the manuscript. Our thanks are particularly due to John Thompson (Liverpool Polytechnic) who made detailed comments on the whole manuscript which helped us to remove many errors. Peter Stoney, Tim Worrall and Ken Cleaver, of the University of Liverpool, kindly read particular sections and made helpful comments which improved the manuscript. Since none of these saw the final manuscript then, as always, the responsibility for any errors lies with the authors. Thanks are also due to Simon Blackman, for help with the graphs and Jenny Holden, for valuable assistance with preparation of the manuscript and the index.

This book is a much revised and expanded version of *Introductory Mathematics for Economists* (Macmillan, 1983) which was originally published by David and Charles (Holdings) Ltd in 1973.

Finally, a word of warning. This book is introductory and intentionally uses simple models. These are not meant to represent the real world and many of the numbers chosen – for example, in supply and demand analysis – and the assumptions imposed are arbitrary and for illustrative purposes only. The world is complicated and it is only by simplifying it that we can hope to gain an understanding of it. But once we do master simple models, more complex and realistic models can be considered.

K.H.
A.W.P