Digital Control

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Digital Control

Fundamentals, Theory and Practice

W. Forsythe and R. M. Goodall

*Department of Electronic & Electrical Engineering
Loughborough University of Technology*

**Macmillan New Electronics
Introductions to Advanced Topics**

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Series Editor's Foreword

The rapid development of electronics and its engineering applications ensures that new topics are always competing for a place in university and polytechnic courses. But it is often difficult for lecturers to find suitable books for recommendation to students, particularly when a topic is covered by a short lecture module, or as an 'option'.

Macmillan New Electronics offers introductions to advanced topics. The level is generally that of second and subsequent years of undergraduate courses in electronic and electrical engineering, computer science and physics. Some of the authors will paint with a broad brush; others will concentrate on a narrower topic, and cover it in greater detail. But in all cases the titles in the Series will provide a sound basis for further reading of the specialist literature, and an up-to-date appreciation of practical applications and likely trends.

The level, scope and approach of the Series should also appeal to practising engineers and scientists encountering an area of electronics for the first time, or needing a rapid and authoritative update.

Paul A. Lynn

Preface

The purpose of this text is to examine the problems inherent in the use of a digital processor for purposes of control, problems both of theory and of practice. Many volumes have already been written with this aim but in our opinion there is room for yet another; there are things that need to be said, important issues that need to be clarified, which current textbooks tend to ignore or to deal with inadequately in one respect or another. The philosophy is not to offer the reader everything we know, but to restrict the information to little more than the essentials. This has two desirable effects: the size and therefore the cost of the book is kept to a minimum and, secondly, the beginner is not called upon to distinguish in his studies between what is and what is not *necessary* for a proper understanding of the subject.

Throughout the book we have attempted to keep the needs of the beginner in mind; the z -transform, for example, is in our experience less understood than its continuous counterpart, the Laplace or s -transform. This cannot be explained purely in terms of the mathematics since the z -transform is the simpler concept. The problem lies, we believe, in the conventional exposition, so the attempt has been made to develop the z -transform on an intuitive basis, to show that the mathematical definition ‘makes sense’. The modified z -transform is dealt with too, since this requires only a slight extension of the idea.

Some trouble is taken to explain the operation of the sampled data loop and the steps needed to construct a suitable mathematical description of it. This entails clarification of such basic concepts as the impulse and an explanation of the meaning and use of the z -transform table. Perhaps surprisingly, it appears to be one of the chief complaints of the beginner that it is difficult to gain a clear perception of these very basic notions.

Methods of designing digital compensation for the sampled data loop are presented, and a number of them discussed in some detail – those that in our estimation the reader will most readily pick up and which will serve for most occasions.

One of the most difficult topics to write meaningfully about is the choice of sample interval, yet it cannot be avoided. It is treated here at some length and is discussed in the context of error generation and filter structure, to both of which it is intimately related.

Many texts attempt to deal with the basics of digital control without giving much attention to the practical issues of implementation: yet there are a number of difficulties that arise and pitfalls that must be avoided when implementing even quite simple controllers. Considerable space is therefore devoted to examination of the fundamental issues that need to be considered when implementing a digital controller: quantisation of the coefficients and internal variables, the implications of this with respect to computational requirements, the hardware and software aspects of the controller itself. In a number of cases both equations and software structures are illustrated by means of program code, and we have chosen to use C because of its applicability to real time applications. We have restricted such coding to the most basic program constructions, and the reader should be able to interpret them without prior knowledge of the language.

The problems of realisation that are dealt with here use examples which are typically encountered in the design of electromechanical servo systems rather than industrial process control. The same realisation problems occur in both types of application [Be88], but they are generally more acute in the former because of the higher speed of operation. The theoretical concepts that are explained underlie control systems of all types.

As well as emphasising what is to be found in the pages that follow, it is necessary to say something about what is not. The most notable omission is perhaps an account of the mathematics of the z -operator. The exclusion of this material is in line with the philosophical approach mentioned above; one can work quite satisfactorily in discrete terms without a knowledge of the more arcane features of the z -domain, provided, that is, the z -transform table is taken as given.

Since it makes no sense to embark on a study of digital control without previous acquaintance with at least the basic ideas of classical control theory, the assumption is made that the reader understands the rudiments of that subject. Given that this is the case, and not very much is presumed, the reader should have little difficulty in grasping the few theoretical concepts that are presented.

Digital control turns out to be computationally very intensive so that the execution of even the simplest ideas entails a great deal of tedious calculation. It helps very much to have available some software package that will largely remove the burden of the computation, and we are willing to supply a copy of our own suite of programs to anyone who wants it. Those interested should write to the address below for details of price (which is very modest) and availability. The software in question performs transformations between the s - and z -domains via the transform table (including the table of modified transforms), the bilinear transformation, and the pole mapping transform described in Appendix E of this book. It is also very desirable to have available some control system design software

to carry out the exercises suggested, or to validate examples quoted; quite a few such packages can now be bought. Many of the plots to be found in these pages were drawn using Simbol 2, from Cambridge Control Ltd. Those readers engaged in teaching, either as teachers or students, might be interested in the *Manual of Solutions* which is again available on request, at the address below.

Finally, we must acknowledge our great debt to Jean Grantham and Christine Sharpe for their invaluable assistance with the production of the present volume. Their good humour and infinite patience are much appreciated.

*Department of Electronic and Electrical Engineering,
Loughborough University of Technology,
Loughborough,
Leicestershire LE11 3TU*

Bill Forsythe
Roger Goodall