Introduction to Nuclear Reactions

A typical experimental area for making measurements on the scattering of a beam of particles accelerated by a cyclotron.
(Photograph courtesy of E. E. Gross, Oak Ridge National Laboratory.)

INTRODUCTION TO NUCLEAR REACTIONS

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SECOND EDITION

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MACMILLAN

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Preface to the Second Edition

The past decade has seen a remarkable growth in the extent and variety of experiments being done on nuclear reactions. The range of energies available has increased by two orders of magnitude. Because of the rapid expansion of heavy-ion science, the number of projectiles in use is very much greater and now includes nuclei as heavy as those of uranium atoms. However, the basic physical principles and techniques involved in the description of these reactions has remained unchanged, although the emphasis may have shifted to some degree. For example, the application of semi-classical ideas became more extensive with the increased interest in heavy-ion reactions, while relativistic considerations became more important as the energy increased.

These advances have resulted in some changes being made in the text, together with some modernisation of the references. Again, the reader is urged to consult review journals and the proceedings of recent conferences for the latest developments. The many experimental examples shown in the figures have not been changed; these were mostly drawn from work done at energies lower than those currently available. Nonetheless, the features illustrated remain relevant at these higher energies.

The opportunity has been taken to correct some misprints, errors, omissions and obscurities that were present in the original. I am indebted to numerous people for bringing many of these to my attention, especially R. C. Johnson, S. Kobayashi and A. G. Tibell.

Oak Ridge, Tennessee, 1990

G.R.S.

Preface to the First Edition

This book is aimed primarily at the undergraduate or beginning graduate student, although I believe others will also find it useful. The specialist from another field may find here a summary of the present situation in our understanding of nuclear-reaction phenomena. The established nuclear physicist may find reading it to be helpful in refreshing his memory about areas in which he is not currently working himself.

Chapters 1 and 2 are intended to provide an overview of the subject which can be readily understood by the novice. They also serve as an introduction to the somewhat more serious remainder. Chapter 3 reviews scattering theory with emphasis on the underlying physical ideas. It also provides schematic *entrées* to the more advanced topics. (There are other excellent texts available which expound these more formal and mathematical aspects of scattering theory.) The discussion is not specific to nuclei, so that Chapter 3 may serve equally well as an introduction to the theory of atomic and molecular collisions. The physical models which have been developed to account for the various aspects of nuclear reaction phenomena are described in more detail in Chapter 4, which is the largest section of the book. I believe that this arrangement enables the book to cater to the needs of a variety of readers without sacrificing any coherence of the presentation as a whole.

Some acquaintance with quantum mechanics is assumed, but not to any great depth. In general, the emphasis here is on the word 'introduction' in the title. There are a number of books and many review articles which treat various parts of the subject in detail and at a higher level; some of these are referred to in the text and are listed in the references at the end of each chapter. Many references are made also to original research papers in the belief that the reader should be encouraged to dip into these other sources of material even if generally they do seem to be more technically advanced than he needs.

Nuclear physics is still very much a living, developing field of study. Consequently any book such as this one is in danger of being obsolescent in some res-

pect as soon as it appears. (This is particularly liable to apply to the descriptions of heavy-ion reactions). This is another reason to urge the reader to supplement the material presented here by resorting freely to the current literature such as review journals and books, and reports of the proceedings of conferences.

The instrumental, experimental and technological aspects of studying nuclear reactions are scarcely mentioned in the present volume; the purpose here is to understand the results of measurements rather than to describe how they are made. There are a number of good books available which address themselves to experimental problems and techniques; we mention in particular *Atomic Nuclei and Their Particles* by E. J. Burge (Oxford University Press, 1977) and, at a more advanced level, the excellent *Techniques in Nuclear Structure Physics* by J. B. A. England (Halsted, New York, 1974).

The first draft of this book was written in the summer of 1970 while the author was a guest of the Aspen Center for Physics, Colorado. I am indebted to the Center for providing a climate so conducive to this achievement. Various sections of the text were read by D. M. Brink, K. T. Hecht and D. K. Scott; I am grateful for their helpful comments. I am also grateful to the many colleagues who have permitted me to reproduce their illustrations here. Finally, I am indebted to Mrs. Althea Tate for efficiently, patiently and cheerfully typing and retyping the manuscript.

Oak Ridge, Tennessee, 1980

G.R.S.