Numbers Through the Ages

**Edited by
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*Reader in the History of Mathematics
Open University*

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Preface

The material of this book originally formed part of the Open University course AM289 *History of Mathematics*.

One fundamental aspect of the history of mathematics deals with the evolution of the concept of *number*, with the representation of numbers by words and symbols, and with the basic methods of calculation which have developed from ancient times to the present day. This aspect is of considerable general interest, and is accessible to the general reader including people who are not mathematically inclined. Since the Open University course was designed for a wide spectrum of students, the topic of 'number' was chosen as a special area of study suitable for non-mathematicians taking the course as an alternative to the *calculus* – an area of historical study requiring considerable mathematical experience.

The original course units which have been edited to form the present work were written by Professor B. L. van der Waerden, Dr Menso Folkerts, Dr E. Neuenschwander, Dr S. H. Hollingdale, and the present Editor, who was also chairman and principal author of the course, and was responsible for the academic editing of the material written by other authors. The units were as follows:

- N1 *Counting I: Primitive and More Developed Counting Systems* (van der Waerden and Flegg)
- N2 *Counting II: Decimal Number Words, Tallies, and Knots* (van der Waerden and Flegg)
- N3 *Written Numbers* (van der Waerden and Folkerts)
- N4 *Written Fractions* (van der Waerden)
- N5 *Methods of Calculation* (van der Waerden and Neuenschwander)
- 11 *Mathematics and Man* – four sections from *Part A* (Hollingdale).

The material from Unit 11 was not part of the 'numbers' option.

In the course of the present editing, exercises and specimen answers have been deleted or have been incorporated into the main text. Statements of objectives and a number of inessential illustrations have also been removed. In some cases the order of presentation has been changed, and in a few instances additional material has been provided by the Editor for completeness or to reflect aspects of recent research.

AM 289 *History of Mathematics* was successfully presented by the Open University for ten years, the final year of presentation being 1985. Since that year there have been many requests received to have aspects of the course made once again available. In presenting this present work it is very much hoped that such requests may be, at least to a significant extent, adequately met.

Particular thanks are due to Professor van der Waerden for his co-operation in the preparation of the original 'numbers' units, and also for readily agreeing to the re-editing of his material to form this book.

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GRAHAM FLEGG