Fundamental Structural Analysis

By the same author

Microcomputer Applications in Structural Engineering (with W. H. Mosley)

Other Macmillan titles of related interest

An Introduction to Engineering Fluid Mechanics

J. A. Fox

Polymer Materials: An Introduction for Technologists and Scientists

Christopher Hall

Prestressed Concrete Design by Computer

R. Hulse and W. H. Mosley

Reinforced Concrete Design by Computer

R. Hulse and W. H. Mosley

Civil Engineering Materials, third edition

edited by N. Jackson

Reinforced Concrete Design, third edition

W. H. Mosley and J. H. Bungey

Strength of Materials, third edition

G. H. Ryder

Surveying for Engineers

J. Uren and W. F. Price

Fundamental Structural Analysis

W. J. Spencer

*Department of Civil Engineering,
Swinburne Institute of Technology,
Hawthorn, Victoria, Australia*

M
MACMILLAN
EDUCATION

© W. J. Spencer 1988

All rights reserved. No reproduction, copy or transmission of this publication may be made without written permission.

No paragraph of this publication may be reproduced, copied or transmitted save with written permission or in accordance with the provisions of the Copyright Act 1956 (as amended), or under the terms of any licence permitting limited copying issued by the Copyright Licensing Agency, 33-4 Alfred Place, London WC1E 7DP.

Any person who does any unauthorised act in relation to this publication may be liable to criminal prosecution and civil claims for damages.

First published 1988

Published by
MACMILLAN EDUCATION LTD
Houndmills, Basingstoke, Hampshire RG21 2XS and London
Companies and representatives throughout the world

British Library Cataloguing in Publication Data
Spencer, W. J.

Fundamental structural analysis.

1. Structures, Theory of

I. Title

624.1'71 TA645

ISBN 978-0-333-43468-0

ISBN 978-1-349-19582-4 (eBook)

DOI 10.1007/978-1-349-19582-4

Diskette copies of 'MATOP' for IBM and IBM-compatible machines are available from the author, from whom further details can be obtained on request. Please address enquiries to:

Dr W. J. Spencer
Department of Civil Engineering
Swinburne Institute of Technology
P.O. Box 218
Hawthorn, Victoria 3122
Australia

Contents

<i>List of Examples</i>	xi
<i>Preface</i>	xiii
1 Introduction to Structural Engineering	1
1.1 The Nature of Structures	1
1.2 Equilibrium and Compatibility	4
1.3 Stress–Strain Relationships and Boundary Conditions	4
1.4 Structural Analysis—An Illustrative Example	5
1.5 Linearity, Stability and Loading	7
1.5.1 Geometric Non-linearity	8
1.5.2 The Stability of Structures	9
1.5.3 Loads on Structures	10
1.6 Scope of the Book—Fundamental Assumptions	11
1.6.1 Sign Conventions	11
Further Reading	12
2 Equilibrium Analysis and Determinacy of Structures	13
2.1 The Equations of Equilibrium	13
2.1.1 Moment of a Force about an Axis	14
2.1.2 Sufficiency of Restraint	15
2.1.3 Determinacy of the Basis of Equilibrium	20
2.2 A Classification of Structures	20
2.2.1 Skeletal Frames	21
2.3 Internal Actions in Structures	21
2.4 Statical Determinacy	22
2.4.1 Equations of Condition	24
2.4.2 Degree of Statical Indeterminacy	25

2.5	Kinematic Determinacy	27
2.5.1	Degrees of Freedom	28
2.5.2	Nodal Degrees of Freedom	28
2.5.3	Structure Degrees of Freedom	29
2.6	Analysis of Statically Determinate Beams and Frames	30
2.7	Analysis of Statically Determinate Trusses	36
2.8	Problems for Solution	41
	Further Reading	43
3	Basic Concepts of the Stiffness Method	44
3.1	Element and Structure Stiffness	45
3.2	Forming the Structure Stiffness Matrix by Direct Multiplication	46
3.3	Solution to Obtain Displacements	51
3.4	Nature of the Structure Stiffness Matrix	53
3.5	Development of the Slope-Deflection Equations	57
3.5.1	The Moment-Area Theorems	58
3.5.2	The Beam Element Stiffness Matrix	61
3.6	Application to Some Simple Beam Problems	65
3.7	Standard Solutions to Beam Problems	69
3.8	Problems for Solution	71
4	The Matrix Stiffness Method—Part 1: Beams and Rectangular Frames	72
4.1	The Analysis of Continuous Beams	72
4.1.1	Forming the Structure Stiffness Matrix	72
4.1.2	Solving for Displacements	76
4.1.3	Element Actions	77
4.1.4	Consideration of Transverse Loads	77
4.2	The Analysis of Rectangular Frames	87
4.2.1	The Column Element Stiffness Matrix	88
4.2.2	Assembly of the Structure Stiffness Matrix	89
4.3	The Direct Stiffness Method	101
4.4	Modification to Element Stiffness Matrices for End Moment Release	106
4.5	Application of the Stiffness Method to Beams and Rectangular Frames	112
4.6	Problems for Solution	117
5	The Moment Distribution Method	119
5.1	An Iterative Solution to a Set of Simultaneous Equations	119
5.2	The Elements of the Moment Distribution Method	122
5.3	Application of the Moment Distribution Method	125
5.3.1	Modification for Pin-ended Elements	131

5.4	Moment Distribution applied to Swaying Rectangular Frames	133
5.4.1	Beam Element Behaviour under Transverse Displacement	133
5.4.2	Frames with One Sway Degree of Freedom	134
5.4.3	Frames with Multi-sway Degrees of Freedom	139
5.5	Problems for Solution	145
	Reference	146
	Further Reading	146
6	The Matrix Stiffness Method—Part 2: Coordinate Transformation	147
6.1	The General Analysis of Trusses	148
6.1.1	The Plane Truss Element	148
6.1.2	Coordinate Transformation for a Truss Element	150
6.1.3	Assembly of the Structure Stiffness Matrix using Truss Elements	152
6.1.4	Solution for Element Actions	154
6.2	The General Analysis of Plane Frames	155
6.2.1	The General Plane Frame Element	155
6.2.2	Coordinate Transformation for a Frame Element	157
6.2.3	Application of Boundary Conditions—Solution for Displacements	157
6.2.4	The Bandwidth of the Stiffness Matrix	171
6.2.5	Frame Elements with End Moment Releases	173
6.3	Composite Structures—Truss and Frame Elements	174
6.4	Problems for Solution	179
	Reference	182
	Further Reading	182
7	The Principle of Virtual Work	183
7.1	Work Concepts	184
7.2	The Principle of Virtual Displacements	185
7.2.1	The Principle of Virtual Displacements applied to a Rigid Body	185
7.2.2	The Principle of Virtual Displacements applied to a Deformable Body	191
7.2.3	A Mathematical Illustration of the Principle of Virtual Displacements	195
7.3	The Principle of Virtual Forces	196
7.3.1	General Application to the Deflection of Frames	198
7.3.2	General Application to the Deflection of Trusses	202
7.3.3	Deflection due to Temperature, Lack of Fit and Support Movements	204
7.4	The Reciprocal Theorems	206

7.5	Proof of the Relationship between the Statics Matrix and the Kinematics Matrix	209
7.6	Problems for Solution	211
	References	213
	Further Reading	213
8	The Flexibility Method of Analysis	214
8.1	Basic Concepts of the Flexibility Method	214
8.1.1	Analysis of Structures with One Degree of Statical Indeterminacy	217
8.1.2	Application to Higher-order Statically Indeterminate Structures	220
8.1.3	Deflection Calculations for Statically Indeterminate Frames	226
8.2	Matrix Formulation of the Flexibility Method	227
8.2.1	Forming the Flexibility Matrix	232
8.2.2	Analysis for Temperature and Support Movement	235
8.2.3	Element Flexibility Matrices	238
8.2.4	Fixed End Actions by Flexibility Analysis	240
8.3	Problems for Solution	242
	Further Reading	244
9	The Approximate Analysis of Structures	245
9.1	Approximate Analysis of Beams and Rectangular Frames	246
9.1.1	Flexural Elements and Points of Inflection	247
9.1.2	Approximate Analysis of a Two-bay Rectangular Frame	251
9.1.3	Approximate Analysis of Multi-storey Rectangular Frames	255
9.2	Bounds on Solutions	260
9.3	Problems for Solution	261
	References	263
10	Application of Computer Programs to Structural Analysis	264
10.1	The Structure of an Analysis Program	266
10.1.1	Data Input	267
10.1.2	Data Output	269
10.2	Modelling of Structures	270
10.2.1	Element Connections	271
10.2.2	Boundary Conditions	272
10.2.3	The Modelling of Non-skeletal Structures by One-dimensional Elements	272

10.3	Influence of the Computer Program on Modelling	273
10.3.1	Additional Effects on Structures	274
10.3.2	The Use of Symmetry	277
	References	278
<i>Appendix A: MATOP (Matrix Operations Program)—User Manual</i>		279
A.1	Introduction	279
A.2	Form of the Program	279
A.3	Operation of the Program	279
A.4	The Command Formats	280
A.5	An Example of the Use of the Program—Solution of Simultaneous Equations	282
A.6	Listing of the Program MATOP	283
<i>Appendix B: Structural Mechanics Students' Handbook—A Manual of Useful Data and Information</i>		298
Part 1		
B1.1	Introduction—Convention	298
Table B1.1	Simple beam moments and deflections (uniform EI)	299
Table B1.2	Some properties of area	300
Table B1.3	Indeterminate beams end moments—transverse loads (uniform EI)	301
Table B1.3A	Indeterminate beams end moments—translation only (uniform EI)	302
Table B1.4	Beam end rotations under transverse load	303
Table B1.5	Standard integrals relating to moment diagrams	304
Table B1.6	Second moments of area	306
B1.2	Summary of the Slope-Deflection Equations	307
B1.3	Use of Table B1.4—Fixed End Moment Calculation	307
Part 2		
B2.1	Introduction—Convention	308
B2.2	Continuous Beam Element	309
B2.3	Continuous Beam Element—LHE-pinned (Moment Release)	310
B2.4	Continuous Beam Element—RHE-pinned (Moment Release)	310
B2.5	Column Element	311
B2.6	Column Element—Base-pinned	311
B2.7	General Plane Frame Element	312
B2.8	Plane Grid Element	313
B2.9	Space Frame Element	314
	Reference	317
<i>Index</i>		319

List of Examples

Example 2.1: Equilibrium Analysis of a Simple Bent	34
Example 2.2: Equilibrium Analysis of a Building Frame	35
Example 2.3: Equilibrium Analysis of a Truss	38
Example 3.1: Analysis of a Truss	53
Example 3.2: Deflections of a Cantilever Beam (1)	67
Example 4.1: Deflections of a Cantilever Beam (2)	79
Example 4.2: Analysis of a Continuous Beam	83
Example 4.3: Analysis of a Rectangular Plane Frame	94
Example 4.4: Frame Analysis using the Direct Stiffness Method	103
Example 4.5: Analysis of a Continuous Beam	109
Example 4.6: Analysis of a Non-swaying Rectangular Frame	114
Example 5.1: Moment Distribution of a Non-swaying Frame	132
Example 5.2: Moment Distribution of a Swaying Frame	136
Example 5.3: Moment Distribution of a Multi-storey Frame	141
Example 6.1: Analysis of a Continuous Beam	160
Example 6.2: Analysis of a Plane Frame	164
Example 6.3: Beam on an Elastic Foundation	177
Example 7.1: Reactions in an Indeterminate Frame	189
Example 7.2: Forces in an Indeterminate Truss	190
Example 7.3: Deflections in a Statically Determinate Frame	199
Example 7.4: Deflections in a Statically Determinate Truss	202
Example 8.1: Flexibility Analysis of a Tied Portal Frame	217
Example 8.2: Flexibility Analysis of a Higher-order Indeterminate Frame	221
Example 8.3: Flexibility Analysis of a Higher-order Determinate Truss	223
Example 8.4: Deflection of a Statically Indeterminate Frame	226
Example 9.1: Beam Analysis	248
Example 9.2: Approximate Analysis of a Two-bay Frame	254
Example 9.3: Approximate Analysis of a Multi-storey Frame	257

Preface

Significant changes have occurred in the approach to structural analysis over the last twenty years. These changes have been brought about by a more general understanding of the nature of the problem and the development of the digital computer. Almost all structural engineering offices throughout the world would now have access to some form of digital computer, ranging from hand-held programmable calculators through to the largest machines available. Powerful microcomputers are also widely available and many engineers and students have personal computers as a general aid to their work. Problems in structural analysis have now been formulated in such a way that the solution is available through the use of the computer, largely by what is known as matrix methods of structural analysis. It is interesting to note that such methods do not put forward new theories in structural analysis, rather they are a restatement of classical theory in a manner that can be directly related to the computer.

This book begins with the premise that most structural analysis will be done on a computer. This is not to say that a fundamental understanding of structural behaviour is not presented or that only computer-based techniques are given. Indeed, the reverse is true. Understanding structural behaviour is an underlying theme and many solution techniques suitable for hand computation, such as moment distribution, are retained. The most widely used method of computer-based structural analysis is the matrix stiffness method. For this reason, all of the fundamental concepts of structures and structural behaviour are presented against the background of the matrix stiffness method. The result is that the student is naturally introduced to the use of the computer in structural analysis, and neither matrix methods nor the computer are treated as an addendum.

Matrix algebra is now well taught in undergraduate mathematics courses and it is assumed that the reader is well acquainted with the subject.

In many instances the solution techniques require the manipulation of matrices and the solution of systems of simultaneous linear equations. These are the operations that the digital computer can most readily handle and they are operations which are built into computer application programs in structural engineering. For the student, however, it is important that these operations are understood, so that it is desirable to have a form of matrix manipulation computer program available. Many programmable pocket calculators currently provide for such operations and there is no doubt that the capacity and speed with which these machines can carry out these tasks will increase with further developments. Some computer languages, notably some versions of BASIC, provide for general matrix manipulation, and scientific library subroutines for handling matrices are provided with other languages such as Fortran. A third possibility is to provide a computer program in the form of a problem-orientated language, with a command structure directly aimed at facilitating the manipulation of matrices. Such a computer program, known as MATOP and developed by the author, is presented as an appendix to the text and used with illustrative examples throughout. The program is not unique and other such programs have been widely available for a number of years.

The text is seen as a first course in structural mechanics or the theory of structures, although it is assumed that students will have done a first course in the more general field of applied mechanics including simple beam theory and stress analysis. The material is probably more than can be covered in two semesters, and indeed it has been delivered over three semesters. The first two chapters outline the fundamental principles and introduce students to the nature of structures and the structural analysis problem. A detailed study of equilibrium and statical and kinematic determinacy is presented in chapter 2. In chapter 3, the foundations of the matrix stiffness method are presented and the ideas of element and structure stiffness matrices are developed. The classical slope-deflection equations are developed from simple beam theory in this chapter, and presented in matrix notation to give the general beam element stiffness matrix.

The matrix stiffness method is further developed in chapter 4, where it is applied to continuous beams and rectangular frames. At this stage coordinate transformation is not introduced and axial deformation of the element is ignored. The approach leads to some powerful applications where the analysis results can be obtained quite rapidly, particularly with the use of the direct stiffness method. It is shown in many instances that the solution is reduced to one of handling matrices of a size that can be adequately dealt with on a pocket calculator.

The moment distribution method has been retained as a useful hand method of analysis and this is detailed in chapter 5, with applications to beams and rectangular frames. The work is closely related to that of chapter 4 and the moment distribution method is shown as a logical variation of

the matrix stiffness method. Chapter 6 returns to the matrix stiffness method to introduce the general stiffness method and coordinate transformation. A wider range of structures is now considered, including composite structures where elements of different types are introduced into the one structure.

A fundamental study of structural analysis must include a reference to the principle of virtual work which is presented in chapter 7. Both the principle of virtual displacements and the principle of virtual forces are considered. The principle of virtual forces, particularly with regard to expressions for the deflection of structures, leads logically into the flexibility method of analysis presented in chapter 8. This provides an alternative approach to the stiffness method and gives a balance to the overall study.

The author is convinced that the general use of computer programs for structural analysis makes demands for greater, rather than less, awareness and understanding of structural behaviour on the part of users. Structural computations must still be checked, results must still be interpreted and engineering judgement must still be exercised. To facilitate this, a chapter on approximate methods of analysis is included (chapter 9). It is presented at this stage since it is felt that approximate methods can only be introduced against a background of general knowledge of structural behaviour.

In a final chapter, some general guidance to computer application programs in structural analysis is presented. Some aspects of modelling of structures are also discussed. Numerous examples are given throughout the text and a common thread is achieved through the use of the program MATOP, details of which are given in an appendix with a program listing in Fortran 77. Much of the data presented throughout the text is collected together in another appendix as a 'Structural Mechanics Students' Handbook'. The significant data here is a collective statement of the element stiffness matrices for various element types.

W. J. Spencer