Business Mathematics

Also by Jim Dewhurst

MATHEMATICS FOR ACCOUNTANTS AND MANAGERS

BUSINESS COST-BENEFIT ANALYSIS

SMALL BUSINESS: Finance and Control (*with Paul Burns*)

SMALL BUSINESS IN EUROPE (*editor with Paul Burns*)

Business Mathematics

Jim Dewhurst

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Contents

<i>Preface</i>	xi
<i>Acknowledgements</i>	xiii
1 Elementary Mathematics	1
1.1 The concept of number	1
1.2 Rounding and truncating	4
1.3 Elementary algebra	5
1.4 Operator precedence	6
1.5 Percentages, discounts and mark-ups	7
1.6 Fees and commissions	9
1.7 Simple interest	10
1.8 Ratios	11
1.9 Exercises	12
2 Computer Number Systems	14
2.1 Introduction	14
2.2 Binary digits	14
2.3 Binary calculations	16
2.4 Computer codes	17
2.5 Octal system	17
2.6 Quintal system	20
2.7 Hexadecimal system	20
2.8 Binary coded decimals	22
2.9 6, 8 and 16 bits	23
2.10 Decimal-to-binary conversion	23
2.11 Exercises	24
3 Exponents, Progressions, Present value	27
3.1 Introduction	27
3.2 Exponential form	28
3.3 Computer arithmetic	29
3.4 Logarithms	30
3.5 Arithmetic progressions	33
3.6 Geometric progressions	34
3.7 Infinite geometric progressions	35
3.8 Compound interest	35

3.9	Frequency of conversion	36
3.10	Present value	37
3.11	Interpolation	38
3.12	Annuities	40
3.13	Annuities for perpetuity	41
3.14	Deferred annuities	42
3.15	Capital investment appraisal	43
3.16	Investment appraisal example	46
3.17	Exponential smoothing	48
3.18	Exercises	50
4	Sets and Relations	58
4.1	Defining a set	58
4.2	Empty set, universal set, subsets	59
4.3	Venn diagrams	60
4.4	Union, intersection, complement, difference	60
4.5	Algebra of sets and duality	62
4.6	Ordered pairs, products	64
4.7	Relations	65
4.8	Exercises	68
5	Matrices, Vectors and Determinants	72
5.1	Matrices	72
5.2	Vectors	73
5.3	Matrix addition and multiplication	74
5.4	Square and identity matrices	77
5.5	Inversion of matrices	78
5.6	Determinants	79
5.7	Inversion of matrices by determinants	80
5.8	Transformation matrices and their use in matrix inversion	82
5.9	Multidimensional arrays	85
5.10	Input–output analysis	86
5.11	Exercises	90
6	Algorithms	93
6.1	Flowchart preparation	93
6.2	Flowchart symbols	93
6.3	Loops	97
6.4	Pseudocode programs	97
6.5	Exercises	101

7	Linear and Higher-power Equations	105
7.1	Use of equations	105
7.2	Linear equations	105
7.3	Gaussian elimination	108
7.4	Matrix inversion method	111
7.5	Iterative method solution of equations	112
7.6	Cubic equations	116
7.7	Power of four	117
7.8	Other iterative methods	118
7.9	Exercises	119
8	Graphs	122
8.1	Introduction	122
8.2	Axes and co-ordinates	122
8.3	Graph of a linear equation	123
8.4	Points lying on either side of a line	125
8.5	Non-linear graphs	127
8.6	The circle	128
8.7	The hyperbola	130
8.8	The exponential function	132
8.9	Graphs and business problems	133
8.10	Economic approach to investment, financing and dividend decision making	137
8.11	Exercises	143
9	Calculus	147
9.1	Differentiation	147
9.2	The product rule	150
9.3	The quotient rule	152
9.4	The function of a function (or chain) rule	152
9.5	The exponential and logarithmic functions	153
9.6	Maxima and minima	153
9.7	Differentials	159
9.8	The integral calculus	159
9.9	Simple integration techniques	160
9.10	Calculation of the area under a curve	162
9.11	Partial differentiation	164
9.12	Total differentials	167
9.13	Three-dimensional maxima and minima	168
9.14	Maxima and minima subject to constraint	170
9.15	Multivariate functions and multiple constraints	171
9.16	Elasticity of demand	172
9.17	Marginal analysis	173
9.18	The economic order quantity model	173
9.19	Exercises	178

10	Probability	186
10.1	Probability theory	186
10.2	Finite probability spaces	187
10.3	Equiprobable spaces	188
10.4	Conditional probability	188
10.5	Independence	190
10.6	Probability and combinatorial analysis	191
10.7	Inventory stock out costs	191
10.8	Risk and uncertainty in business forecasting and decision making	192
10.9	Markov chains	203
10.10	Absorbing states	206
10.11	Combinatorial analysis	208
10.12	Permutations	209
10.13	Combinations	211
10.14	Binomial theorem	212
10.15	Exercises	212
11	Regression Analysis	220
11.1	Introduction	220
11.2	Simple regression	221
11.3	Formulae for the line of best fit or regression line	223
11.4	Application of the formulae for the line of best fit	224
11.5	Prediction and confidence limits	227
11.6	Multiple regression	228
11.7	Interpretation of the statistical data	232
11.8	Application of the multiple regression formulae	236
11.9	Extensions of the regression model	237
11.10	Dummy variables	238
11.11	Business uses of regression analysis	239
11.12	Discriminant analysis	240
11.13	Z score analysis	242
11.14	Exercises	243
12	Linear Programming (<i>Robert Ashford</i>)	246
12.1	Introduction	246
12.2	A simple problem	246
12.3	Some basic concepts	248
12.4	An algebraic approach	251
12.5	The simplex method	253
12.6	The simplex tableau	259
12.7	Exercises	264

13 Sensitivity Analysis, Duality and the Transportation	
Algorithm (Robert Ashford)	270
13.1 Sensitivity analysis or ranging	270
13.2 Duality	275
13.3 The transportation algorithm	279
13.4 Integer programming	290
13.5 Exercises	290
 <i>Appendix A Tables</i>	 299
<i>A.1 Logarithms</i>	299
<i>A.2 Present value tables</i>	301
<i>A.3 Annuity tables</i>	302
<i>A.4 Probability tables</i>	303
<i>Appendix B Solutions to Exercises</i>	304
<i>Appendix C Worked Answers</i>	310
<i>Further Reading</i>	328
<i>Index</i>	329

Preface

This book is intended for those students and managers whose basic knowledge of mathematics can best be described as being broadly equivalent to the (old!) 'O' level.

The approach used in this book is based partly on 'traditional' maths and partly on modern maths. This reflects the view of the author that both have advantages, and that acting together they can provide a very powerful base. This approach is also desirable for other, more pragmatic reasons. First, readers will have widely differing mathematical backgrounds; second, many professional, managerial, accountancy and banking examinations require an understanding of both these approaches. Further, most techniques these days will be implemented with the aid of a suitable computer program. This book may therefore be seen as being based on the integration of *traditional maths* with *modern maths* through the technique of *computers*.

The material has been divided into thirteen chapters and they have been written so that they are to an extent independent of each other.

Each chapter starts with the relevant mathematics and at the end covers appropriate *applications* to the business situation. There are exceptions to this, since sometimes the business application is more suitably dealt with in the body of the text. An example of this is in Chapter 13, where the transportation algorithm is an integral part of the mathematical text and is treated as such.

Answers are given for most of the quantitative questions (Appendix B); in addition fully-worked answers (the author's responsibility) have been provided for some questions (Appendix C). The intention here has been to provide one of these worked answers for a typical examination question in each of the main areas of the subject.

Except where necessary for a proper appreciation of the business techniques *this book does not concern itself with statistics*. Plenty of books have been written solely on this subject. There have, however, been some 'demarcation' problems! These have been particularly difficult in the general area of what might be described as 'mathematically-based operational research methods'. In order to help bridge this gap a short selected bibliography of books in this area has been provided. Because of their practical and examination importance regression analysis and linear programming have been included.

Chapters 12 and 13 have been written by Robert Ashford (of the School of Industrial and Business Studies, University of Warwick) who is a leading authority in the field of linear programming.

Many readers will certainly be working for university, professional, and other examinations. This book covers the mathematical requirements for the examinations of the Chartered Institute of Cost and Management, the Chartered Institute of Public Finance and Accounting, the Institute of Banking, the Chartered Insurance Institute, and the British Institute of Management.

Sometimes, in this book, sexist words and phrases may be used, though hopefully only where general usage sanctions it.

Finally, my thanks to Louise Harris, Shirley Clarke, Robert Ashford, Phillip Moon and Graham Jones.

J.D.

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