An Introduction to
Cost and Production Functions

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The Econometric Study of the UK (editor)

Production Functions

Topics in Applied Macroeconomics (editor)

The Economics of Co-determination (editor)

Perspectives on Inflation: Models and Policy (editor)

AN INTRODUCTION TO COST AND PRODUCTION FUNCTIONS

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Contents

3 **Elasticity, Efficiency and the Theory of the Firm 53**

4 **The Cobb–Douglas Function 76**

5   **The CES Function   92**

6   **The Translog Function   105**

7   **Technological Progress   118**

Authors' Preface

There can be little doubt that production functions, and their associated cost functions, form an integral part of an enormous range of economic theory. In microeconomics, production functions underlie the supply side of markets, generate production possibility frontiers, offer an explanation of income distribution and yield factor demand functions. Production functions are also central to the theory of economic growth and to investigations into the rate of technological progress. In international trade, production functions are used to provide a rationale for product and factor movements across national boundaries. In macroeconomics, production functions lie behind aggregate supply functions, aggregate labour demand functions and form the link between output and the consequent employment. Even the demand side can make use of production functions. The functional forms used for utility functions are often 'borrowed' from production theory. Indeed the production model has been transferred wholesale to model 'demand'.

It is clear from this somewhat impressive list that a knowledge of production functions is a useful if not essential part of being an economist.

The kind of production function most widely used is the so-called 'neoclassical' production function. Neoclassical functions can take a host of forms (as we shall see in the following chapters) but they all have three fundamental characteristics. First they represent ways in which labour, capital and land can be combined to produce goods. Second, they assume that capital is a separate, independent input

directly comparable with labour and land. And third, they focus attention on the production possibilities and decisions within processes, firms or industries.

Many of the results which spring from these functions, when applied to the various aspects of economics listed above, rely on these implicit assumptions. They are not without their critics.

A slightly different approach derives from general equilibrium models and focuses attention on *inter*-firm relationships of production. The Leontief production model is perhaps the best known example of this approach. Each sector uses capital and labour but also needs inputs from other sectors. The 'Food, Drink and Tobacco' sector, for example, buys materials from Agriculture, and Agriculture buys from Chemicals and so on. There is no substitution among inputs and so the inputs into each sector are usually assumed to be determined simply by the output of that sector. This approach is known as *input–output analysis* and concentrates on the interdependence of industries, firms and processes rather than choice of techniques within an industry, firm or process.

The input–output approach is widely used in planning models where the 'balance' among industries is important. And input–output is largely regarded as an alternative to neoclassical functions rather than a contradiction of them.

A third approach, the 'classical' production model, does however directly contradict the 'neoclassical' model. This approach is like the input–output approach in that attention is focused on inter-industry relations. In this classical model, however, one of the sectors produces capital goods – capital is explicitly recognised as a *produced* input. Whereas land and labour are 'original' inputs, capital is also an output. This modification is sufficient to render many of the 'standard' results of neoclassical production theory invalid. It is no longer unambiguously true, for example, that increasing the interest rate *vis-à-vis* the wage rate will induce capital saving. Sraffa (1972) has shown that it is possible for a particular man/machine combination to be used at low rates of interest, fall into disuse as interest rates rise and then, as they rise still further, switch back into operation again. This result would not be possible in a neoclassical model of production. As interest rate rises less and less capital is used and more and more labour so that the capital–labour ratio continues to fall for all increases in interest rate.

According to this classical model it is simply not possible simul-

taneously to determine factor and product prices as it is in the neoclassical world. It is necessary first to specify the wage/interest ratio and from this the choice of technique and product prices are found.

The so-called 'capital controversy' has been widely covered in the literature (Harcourt (1972), Kregel (1976) and cf. Bliss (1975)) and is rather beyond the scope of an introductory text such as this. Whatever merits or flaws the neoclassical production model may have it is indisputably the dominant model and economists require to have some understanding of it.

It is not our intention here to stress the controversies or to offer a comprehensive account of the various applications of production theory. Our aim is simply to bring together in one volume the principal neoclassical approaches to production and to compare and contrast their properties.

Production functions imply particular cost functions, often the 'self-dual' of the production function. These cost functions are sometimes of interest in their own right but are sometimes used as more tractable alternatives to production functions. For these reasons we introduce, where appropriate, the cost functions associated with each production function. We have tried to keep the inevitable mathematics to a fairly simple level and have consigned the more esoteric points to appendices which can be ignored by the general reader. Each chapter begins with fairly simple concepts and becomes progressively more difficult. Some students may find that the early parts of the chapters are all that is required.

There are two general chapters: one on estimating production functions and one on technological progress. These are included merely to indicate to the student some of the difficulties and some possible solutions which have been discussed in the literature.

We have, in short, tried to provide a rigorous yet accessible introduction to the principal aspects of cost and production functions.

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Symbols

The following symbols are used throughout this book:

q = Quantity of *firm output*.
Q = Quantity of *industry output*.
$v_i, \ldots, v_n$ = *Firm inputs* No $1, \ldots, n$.
$V_1, \ldots, V_n$ = *Industry input*.
K = Quantity capital (same for firm and industry).
L = Quantity labour (same for firm and industry).
E = Quantity energy (same for firm and industry)
t = time.
P = Prices of output (same for firm and industry).
$P_1, \ldots, P_n$ = Prices of inputs.
π = Profits.
ε = Elasticity of scale.
ε_c = Elasticity of cost (with respect to production level).
σ = Elasticity of substitution (in a two-factor production model)
σ_{rs} = Allen partial elasticity of substitution between inputs r and s.
E_j = Price elasticity of demand for factor j.
S_j = Cost-share for factor j.
E_{ij} = Cross-price elasticity of demand for factor i.
C = (Total) cost of factors.
f_i = partial derivative of function $f(\ )$ w.r.t. factor i
ξ_j = Input coefficient for factor j (i.e. input per unit of output)
U = Error term
$A, B, a, b, \alpha, \beta, \gamma, \lambda, \theta, \delta$ are used as parameters.

A dot ($\cdot$) above a symbol (e.g. K) indicates *rate of growth*.
A line above a symbol (e.g. $\bar{q}$) indicates any arbitrarily fixed level.
A hat or a star above a symbol (e.g. $\hat{q}$ or $\overset{*}{q}$) indicates some special level (e.g. the cost-minimising level).

Abbreviations

The following abbreviations are used throughout this book.

SR = Short Run
LR = Long Run

TC = Total Cost
FC = Fixed Cost
VC = Variable Cost
MC = Marginal Cost
AC = Average Cost
ATC = Average Total Cost (= AC)
AVC = Average Variable Cost
AFC = Average Fixed Cost
LRAC = Long Run Average Cost
SRAC = Short Run Average Cost

TR = Total Revenue
MR = Marginal Revenue
AR = Average Revenue

TP = Total Product
AP = Average Product
MP = Marginal Product

RTS = Returns to Scale
CRTS = Constant Returns to Scale

DRTS = Decreasing Returns to Scale
IRTS = Increasing Returns to Scale

CD = Cobb–Douglas (Production Function)
CES = Constant Elasticity of Substitution (Production
    Function)