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Applied Elasto-plasticity of Solids

T. Z. Blazynski

*Department of Mechanical Engineering
University of Leeds*

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Preface

Exploitation of the possibilities offered by elasto-plastic design – allowing the manipulation of an in-built system of residual stresses – has reached sufficient practical importance to be taught, where appropriate, in the final undergraduate year of various mechanical, civil and production engineering courses and to be included in some M.Sc. syllabuses.

Although the respective areas of elasticity, in particular, and plasticity are adequately covered by a selection of monographs and texts updated in recent years, the area of elasto-plasticity – concerned with the relationship and interaction between elastic and plastic deformations of the same order of magnitude – appears to be somewhat neglected. This book is intended to bridge that gap by explaining how the two regimes of material behaviour are linked in theory and by applying the concepts and principles so established to a variety of basic engineering components to which purely analytical methods of calculation can be applied.

The book is divided into two parts. Chapters 1 to 3 serve as a ‘refresher course’ on the fundamentals of material behaviour with an analytical description of it and the remaining chapters are concerned with the application of this knowledge to the problems of bending, torsion and axisymmetrical pressure and rotational systems.

To bring the contents completely up to date, discussions of operational, dynamic, industrial techniques are included, although it is recognised that in some undergraduate courses these may not be regarded as necessary. Even so, it is felt that as optional reading, they provide a useful insight into the methods of analytical treatment that are currently available.

Relevant chapters, mostly of the application type, contain a number of carefully selected, solved examples to illustrate the appropriate methods of solution and also a number of unsolved problems, with answers provided, to give the reader an opportunity to practise the expertise acquired.

The mathematical treatment of the subject is entirely within the scope of undergraduate teaching and does not require the knowledge of techniques more advanced than this level suggests. Since the S.I. system is now more generally accepted, numerical examples are given accordingly but a short FPS conversion table is provided for the reader less familiar with its use. Lists of further reading

PREFACE

that either widen the information provided or furnish more detailed treatment of some of the topics are appended to the relevant chapters.

Many of the examples used are taken from past examination and tutorial papers and in this respect I gratefully acknowledge the contributions made to chapters 6, 7 and 8 by Professor B. N. Cole and my other colleagues. Considerable use has been made of the published work of Professor W. Johnson whose permission to include it in the text is much appreciated.

Finally, my thanks go to my wife for her patience during the long period of writing and also for her help with the preparation of the manuscript.

T. Z. BLAZYNSKI
Leeds, 1983

Principal Notation

a	internal radius
A	cross-sectional area, a constant
b	breadth, outer radius
C	a constant
d	diameter
D	diameter, a constant
e	engineering strain
E	pure strain tensor, Young's modulus
f	volume fraction
F	force
G	modulus of rigidity
h	height, thickness
H	enthalpy
I	stress invariant
J	strain invariant, polar moment of area
k	yield stress in shear, Boltzmann's constant
K	bulk modulus, ratio b/a
l	direction cosine
L	length
m	direction cosine
M	moment
n	direction cosine, power index, number of cycles
N	normal component of force
p	pressure
P	force
q	load per unit length
r	radial distance
R	radius of curvature
s	stress component
S	stress tensor
t	time
T	temperature, torque
u	displacement

PRINCIPAL NOTATION

U	energy
v	velocity, displacement
V	volume
w	displacement
W	work done, force
W^e	elastic work
W^p	plastic work
Y	yield stress in simple tension
α	linear coefficient of thermal expansion
γ	shear strain
ϵ	natural, logarithmic strain
ϵ^e	elastic strain
ϵ^p	plastic strain
$\bar{\epsilon}$	generalised strain
$\dot{\epsilon}$	strain rate
μ	coefficient of friction
ν	Poisson's ratio
ρ	density
σ	stress
$\bar{\sigma}$	generalised stress
τ	shear stress

Subscripts

c	interface
E	elastic
H	homogeneous
I	redundant
i	tensorial notation, inner
j	tensorial notation
L	loading phase
m	mean
N	normal
o	initial, nominal, outer
P	plastic
r, θ, z	co-ordinates
R	residual
S	shear
T	total
U	unloading phase
V	volumetric
x, y, z	co-ordinates
Y_1	at first yield
Y_2	at second yield
$1, 2, 3$	principal directions

Conversion to S.I. Units

Length

$$\begin{aligned} 1 \text{ in.} &= 25.4 \text{ mm} = 0.0254 \text{ m} \\ 1 \text{ ft} &= 0.3048 \text{ m} \end{aligned}$$

Area

$$\begin{aligned} 1 \text{ in.}^2 &= 6.4516 \times 10^{-4} \text{ m}^2 \\ 1 \text{ ft}^2 &= 0.0929 \text{ m}^2 \end{aligned}$$

Volume

$$\begin{aligned} 1 \text{ in.}^3 &= 16.387 \times 10^{-6} \text{ m}^3 \\ 1 \text{ ft}^3 &= 0.0283 \text{ m}^3 \end{aligned}$$

Mass

$$1 \text{ lb} = 0.4536 \text{ kg}$$

Density

$$1 \text{ lb/in}^3 = 27\,679.9 \text{ kg/m}^3$$

Moment of inertia

$$1 \text{ lb ft}^2 = 0.04214 \text{ kg m}^2$$

Force

$$1 \text{ lbf} = 4.4482 \text{ N}$$

Moment

$$1 \text{ lbf ft} = 1.3558 \text{ Nm}$$

CONVERSION TO S.I. UNITS

Stress or pressure

$$1 \text{ lbf/in.}^2 = 6894.76 \text{ Pa}$$

$$1 \text{ tonf/in.}^2 = 15.4443 \text{ MPa}$$

Energy

$$1 \text{ ft lbf} = 1.3558 \text{ J}$$