Fundamentals of Mechanical Vibrations

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For my Mother and Father

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Preface

A knowledge and appreciation of mechanical vibrations are fundamental to wide areas of physics, engineering and architecture. Naturally occurring vibrations are of interest to geophysicists, to civil engineers and to medical physicists. Guarding against the effects of excessive vibrations is a prime concern of design engineers, architects, civil engineers, structural and mechanical engineers, instrumentation physicists and biomedical engineers. The design and construction of vibrating machines is often the aim of mechanical engineers and acoustic engineers. The measurement and interpretation of vibrations are important tasks for maintenance engineers and physicists.

The aim of this book is to provide the basic groundwork in mechanical vibrations for these various disciplines. Practice in problem-solving is probably the single most powerful mechanism for learning and therefore it is strongly recommended that the student attempt all of the problems appended to each chapter. The value of the text will be greatly enhanced if its study is accompanied by a course of related laboratory experiments. Each chapter has a list of suggested laboratory exercises relating to the material in that chapter. At the end of each chapter there is a brief list of books for further reference. Whenever possible, the student should consult these in order to supplement and expand on the coverage in this text, to obtain alternative insights, to delve deeper into specific topics and to find additional problems.

Students embarking on this book should have the following prerequisites: differential and integral calculus, including partial differential equations; physics to approximately first-year college level; A.C. electricity, which could be studied concurrently; mechanical properties of materials, which could also be studied concurrently.

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MATTHEW HUSSEY

List of Symbols

A	area	m^2
	amplitude constant	—
a	amplitude constant	—
	radius	m
α	coefficient of decay or decay constant	$1/\text{s}$
	attenuation constant	$1/\text{m}$
B	amplitude constant	—
	bulk modulus of elasticity	Pa
b	damping coefficient	N s/m or kg/s
	amplitude constant	—
β	propagation or wavelength constant	$1/\text{m}$
C	spring constant	N/m
	amplitude constant	—
	electrical capacitance	F
	inverse of compliance per metre of tube wall	Pa/m^2
c	amplitude constant	—
	spring constant	N/m
	speed of propagation	m/s
	Young's modulus of elasticity	N/m^2 or Pa
D	dielectric displacement	C/m^2
d	distance	m
	amplitude constant	—
	piezoelectric constant	C/N or m/V
δ	central deflection of beam	m
	logarithmic decrement	—
	Dirac delta function	—
	non-linear constant for spring	—
Δ	ΔL , increment of L	—
E	Young's modulus of elasticity	N/m^2 or Pa
	energy	J
	electric field strength or potential gradient	V/m
e	piezoelectric constant	C/m^2 or N/V m
	eccentric radius	m
ϵ	dielectric constant	C/m V
F	force	N
	Fourier transform	—

f	frequency	Hz
	instantaneous force	N
	general function	—
φ	angular displacement	rad
	phase angle	rad
Φ	maximum angular deflection	rad
G	force divided by mass	m/s^2
	Gibbs' free energy	J
	amplitude constant	—
g	acceleration due to gravity	$= 9.81 \text{ m/s}^2$
	instantaneous force divided by mass	m/s^2
	piezoelectric constant	m^2/C or V m/N
	mass ratio in dynamic vibration absorber	—
γ	end deflection of cantilever	m
	phase angle	rad
	ratio of specific heats of gas	—
	complex propagation constant	$1/\text{m}$
H	heat	J
	system transfer function	—
	enthalpy	J
	amplitude constant	Pa
$\dot{H}$	rate of heat production	J/s
h	height	m
	impulse response	—
	piezoelectric constant	N/C or V/m
	hysteretic damping coefficient	N/m
η	coefficient of viscosity	kg/s m^2
I	mass moment of inertia	kg m^2
	area moment of inertia	m^4
	electrical current	A
i	instantaneous electrical current	A
$\dot{i}$	rate of change of electrical current	A/s
J	Bessel function	—
	electrical current density	A/m^2
j	$\sqrt{-1}$	—
K	constant	J
	integer	$= 1, 2, 3, \dots$
k	constant	—
	integer	$= 0, 1, 2, \dots$
	coupling coefficient of piezoelectric material	—
κ	mass per unit length of string	kg/m
	compliance or compressibility	m^2/N
	non-linearity constant of spring	—

L	angular momentum	$\text{kg m}^2 \text{ rad/s}$
	length	m
	electrical inductance	H
	Laplace transform	—
$\mathcal{L}$	Laplace transform operation	—
$\mathcal{L}^{-1}$	Inverse Laplace transform operation	—
l	length	m
	integer	$= 0, 1, 2 \dots$
λ	wavelength	m
M	mass	kg
	bending moment or torque	N m
m	integer	$= 0, 1, 2 \dots$
μ	coefficient of static friction	—
	mass per unit area of membrane	kg/m^2
Mom	linear momentum	kg m/s
N	Bessel function	—
n	shear modulus of elasticity	$\text{N/m}^2 \text{ or Pa}$
	integer	$= 0, 1, 2 \dots$
P	power	W
	pressure	Pa
	amplitude of mode displacement	m
	distribution function	—
	protection factor	—
p	constant	m
	instantaneous power	W
	amplitude constant	—
	mode displacement	m
	instantaneous pressure	Pa
	initial displacement	m
	probability density	—
$\dot{p}$	rate of change of instantaneous pressure	Pa/s
π	constant	$= 3.14159\dots$
ψ	phase angle	rad
	mode co-ordinate	$\text{m kg}^{1/2}$
	angular frequency	rad/s
Ψ	amplitude of mode co-ordinate	$\text{m kg}^{1/2}$
Q	quality factor or value	—
	$2 \times$ spectral density function	—
$\dot{Q}$	flow rate	m^3/s
q	constant	m
	instantaneous electrical charge	C
	damping ratio	—
	amplitude constant	—
	initial velocity	m/s
$\dot{q}$	instantaneous flow rate	m^3/s

R	electrical resistance	Ω
	radial distance	m
	universal gas constant	= 8.31 J/mole K
r	radius	m
	radial axis	—
	ratio of frequency to natural frequency	—
ρ	density	kg/m ³
	resistivity	Ω m
S	strain	—
	spectral density function	—
s	circumferential displacement	m
	transform parameter	—
	ratio of third to first harmonic amplitudes	—
$\dot{s}$	circumferential velocity	m/s
$\ddot{s}$	circumferential acceleration	m/s ²
σ	force per unit length of periphery of membrane	N/m
	entropy	J/K
	standard deviation	—
T	stress	N/m ² or Pa
	period of oscillation or of periodic function	s
	transmissibility	—
t	time	s
	temperature	°C
τ	time constant or relaxation time	s
	duration of pulse	s
	time variable	s
ϑ	angular displacement	rad
	angle	rad
	angular axis in cylindrical co-ordinates	—
	absolute temperature	K
$\dot{\vartheta}$	angular velocity	rad/s
$\ddot{\vartheta}$	angular acceleration	rad/s ²
Tr	torque or turning moment	N m
U	step function	—
	internal energy	J
V	volume	m ³
	electrical potential difference	V
v	instantaneous potential difference	V
w	distance	m
ω	angular velocity or angular frequency	rad/s
Wk	work	N m or J
X	Laplace transform of x	—
x	axis	—
	displacement along x -axis	m
	change in length of spring	m

$\dot{x}$	velocity in x -direction	m/s
$\ddot{x}$	acceleration in x -direction	m/s ²
ξ	particle displacement along x -axis	m
$\dot{\xi}$	particle velocity along x -axis	m/s
$\ddot{\xi}$	particle acceleration in x -direction	m/s ²
Y	mechanical compliance	s/kg
y	axis	—
	displacement along y -axis	m
	distance	m
$\dot{y}$	velocity along y -direction	m/s
$\ddot{y}$	acceleration along y -direction	m/s ²
Z	mechanical impedance	kg/s or N s/m
	electrical impedance	Ω
z	axis	—
	displacement along z -axis	m
$\dot{z}$	velocity along z -axis	m/s
$\ddot{z}$	acceleration along z -direction	m/s ²
ζ	particle displacement along z -axis	m
$\dot{\zeta}$	particle velocity in z -direction	m/s
$\ddot{\zeta}$	particle acceleration in z -direction	m/s ²