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Mathematics for Computing

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To Mary and Sheila

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Preface

The problems that early computers solved were mostly mathematical. Since then, as everyone knows, the use of computers has greatly diversified such that today the majority of applications are non-numerical. Thus, while it is immediately clear that a good mathematical background is essential for the obviously mathematical areas of computing, such as numerical techniques, computer simulation or the theory of computation, the question arises as to whether mathematics is now important in computing as a whole. We believe the answer to this question to be an unqualified yes. The last decade or so has seen the development of computing from little more than a 'bag of tricks' into a science. As in any scientific or engineering discipline, mathematics is the medium through which the underlying concepts and principles of computer science may be understood. Mathematical notation is now found in virtually every branch of computing, from the mathematical areas mentioned above, through compiling techniques, data structures and algorithm design and analysis, to data base systems. Without the ability to appreciate and apply mathematical concepts and techniques, the aspiring computer scientist cannot hope to grasp the fundamental principles of computing, principles that will still be relevant even if the particular programming skills that he has learnt become obsolete.

This book is designed for two types of user. The first is the student starting tertiary education in computing, who will need to develop a reasonable mathematical maturity in order to cope with the use of mathematical notation in subsequent computer science courses. We assume that no such student will be without a reasonable 'A'-level (or equivalent) in mathematics. The book provides a basis for a course equivalent to about one-third of the first year of study in a degree programme.

The second category of reader for whom the book is designed is the practising computer scientist who needs a reference book on his shelf to which he can go when he needs a definition and an example of some concept only vaguely remembered. For this reason, after most definitions in the book, one or more examples are given to illustrate the new term.

While the material in this book is essentially traditional mathematics, it has been given a computer science flavour through the use of algorithms.

The algorithm is the core concept in computing, but one which normally has little place in traditional mathematics. Nevertheless, an algorithm is often the best way of describing well-known mathematical techniques.

Since this book is intended primarily to be of use on a general first mathematics course in a computer science degree programme, it is not constrained to just discrete mathematics. We believe that calculus contains a wealth of results with applications in computing, and for this reason a substantial amount of calculus is presented in the third chapter. Although this chapter is called 'Calculus' it does, in fact, contain some finite mathematics. In particular, the section on series includes both the finite and the infinite cases.

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