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MASTERING MATHEMATICS

O. & J. PERRY

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NOTATION

$\{a, b, c, \dots\}$	the set of $a, b, c, \dots$
$:$	such that
$\in$	is an element of
$\notin$	is not an element of
$n()$	the number of elements in the set of
ϕ	the empty (null) set
$\mathcal{E}$	the universal set
$\cup$	union
$\cap$	intersection
$\subset$	is a subset of
A'	the complement of the set A
N	the set of natural numbers
Z	the set of integers
R	the set of real numbers
PQ	operation Q followed by operation P
$f: x \rightarrow y$	the function of mapping the set X into the set Y
$f(x)$	the image of x under the function f
f^{-1}	the inverse of the function f
fg	the function f of the function g
$\text{---} \circ \text{---} \circ \text{---}$	open interval on the number line
$\text{---} \bullet \text{---} \bullet \text{---}$	closed interval on the number line
$\{x: -2 < x < 7\}$	the set of values of x such that . . .
$\Rightarrow$	implies that
$\Leftrightarrow$	implies and is implied by
$=$	is equal to
$\equiv$	is identically equal to
$\approx$	is approximately equal to
$\neq$	is not equal to
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to
$ $	the unsigned part of a signed number, that is the modulus
∞	infinity

PREFACE

In writing this book, we have assumed that you know at least the counting numbers and the addition and multiplication tables up to 9 by 9. The language of mathematics contains a number of specialised words, and these are printed in *italics* the first time they occur in the text. Some of them are new words, such as *logarithm*, *calculus*, *cosine*, which have been invented by mathematicians in the past; others are ordinary English words such as *operation*, which have a special meaning in mathematics. While it is not essential to your understanding of the subject, the use of the correct words and symbols makes communication between mathematicians easier, and it is worthwhile trying to learn the language.

The book covers a very wide field and so the introduction to each section is necessarily concise; you should work through and understand the examples in a section of the work before attempting the exercises that follow it. The use of an electronic calculator will save time and enable you to spend longer on studying the basic mathematical ideas and on practising with past examination papers. Remember that the use of calculators is not allowed in all examinations.

We hope you will enjoy working with this book and that it will bring you success.

Joyce Perry

Owen Perry.