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Structural Theory and Analysis

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Second Edition

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CONTENTS

<i>Preface</i>	ix
1 Plane statics	1
1.1 Introduction	1
1.2 Equations of equilibrium	2
1.3 Supports, reactions and free-body diagrams	2
1.4 Stability and determinacy of reactions	4
1.5 Calculation of reactions	5
1.6 Equation of condition	8
1.7 Principle of virtual work	9
1.8 Shear force and bending moment	11
1.9 Relations between load, shear force and bending moment	15
1.10 Principle of superposition	16
1.11 The force polygon	20
1.12 The funicular polygon	21
1.13 Graphical determination of reactions	22
1.14 Graphical construction of bending-moment diagrams	23
1.15 Graphics applied to a three-pin arch	25
1.16 The differential equation for a vertical load funicular polygon	27
1.17 Introduction to influence lines	28
1.18 Influence lines for reactions, shear force and bending moment	29
1.19 Loading systems	30
1.20 Application of virtual-work methods to influence lines	32
1.21 Multi-load systems	35
1.22 Influence lines for girders with floor beams	39
1.23 Influence line for three-pin arch	40
2 Statically determinate structures	45
2.1 Simple plane trusses	45
2.2 Stability and determinacy	46
2.3 Resolution at joints	48
2.4 Graphical method	50
2.5 Method of sections	51

2.6	Compound trusses	53
2.7	Complex trusses	53
2.8	Virtual work	55
2.9	Statically determinate space structures	57
2.10	Tension coefficients	58
2.11	Rigid jointed structures	60
2.12	Influence lines for statically determinate trusses	66
3	Elementary elasticity, plasticity and bending of beams	73
3.1	Sign conventions	73
3.2	Stress	74
3.3	Strain	76
3.4	Relations between stress and strain	78
3.5	Composite rods with axial tension or compression	80
3.6	Composite rods loaded beyond the yield point	83
3.7	Principal stresses in two dimensions	85
3.8	The Mohr circle diagram for stress	89
3.9	Principal strain in two dimensions	91
3.10	The Mohr circle diagram for strain	93
3.11	Relationships between elastic constants	95
3.12	Strain energy	99
3.13	Pure bending of beams	100
3.14	Bending of a composite beam	105
3.15	Bending of unsymmetrical sections	107
3.16	Strain energy due to bending	110
3.17	Combined bending and axial force	111
3.18	Plastic bending of beams	112
3.19	Combined plastic bending and axial force	118
4	Torsion and shear effects	122
4.1	Torsion of a circular cross-section	122
4.2	Strain energy due to torsion	125
4.3	Combined torsion, bending and axial force	126
4.4	Plastic torsion of circular rods	126
4.5	Shear stresses in beams	128
4.6	Shear-stress distribution in flanges	133
4.7	Shear centre	135
4.8	Torsion of thin-walled tubes	136
4.9	Torsion of a thin rectangular section	138
5	Deflection of beams	142
5.1	Introduction	142
5.2	Deflection by direct integration	143
5.3	Deflections using singularity functions or the Macaulay method	145
5.4	Moment-area methods	150
5.5	Use of standard cases	155
5.6	Deflections due to shear	157

6 Virtual work and energy methods	161
6.1 Introduction	161
6.2 Strain energy and complementary energy	161
6.3 Flexibility coefficients	163
6.4 Maxwell's reciprocal theorem	165
6.5 The flexibility approach	166
6.6 Kinematic indeterminacy	169
6.7 The stiffness approach	169
6.8 The method of real work	172
6.9 Method of virtual work	174
6.10 Virtual work applied to statically determinate systems	175
6.11 Energy theorems and their application to statically determinate structures	181
6.12 Williot-Mohr displacement diagram	189
6.13 Virtual work applied to a statically indeterminate truss	192
6.14 Engesser's theorem of compatibility	196
6.15 Trusses with several redundants	199
6.16 The trussed beam	202
6.17 Virtual work and energy methods applied to frames	203
6.18 Ring and arch problems	207
6.19 Redundant trusses using the stiffness approach	213
7 Moment distribution and slope deflection	218
7.1 Moment distribution	218
7.2 Sign convention	219
7.3 Stiffness and carry-over factor	219
7.4 Distribution factor	221
7.5 Fixed end moments	221
7.6 Examples	224
7.7 Modified stiffness	226
7.8 Deflection of supports and sidesway	231
7.9 Frames with inclined members	237
7.10 Rectangular multi-storey frames	239
7.11 The slope-deflection equation	240
8 Stiffness and flexibility methods	250
8.1 Introduction	250
8.2 Formulation of simple stiffness matrices	250
8.3 Pin-jointed plane trusses	254
8.4 Example of a pin-jointed redundant truss	258
8.5 General outline of the stiffness approach	264
8.6 Stiffness matrix for a space structure member	266
8.7 The space truss	270
8.8 Continuous beams	272
8.9 The plane truss	275
8.10 Grillages	280
8.11 Special cases	281
8.12 Outline of the flexibility method	281

8.13 Member flexibility matrix	285
8.14 Example of a pin-jointed redundant truss	286
8.15 Example of a plane frame	290
8.16 Choice of stiffness or flexibility approach	295
9 Influence lines for statically indeterminate beams	300
9.1 Beams with two spans	300
9.2 Betti's reciprocal theorem	302
9.3 Applications of Betti's theorem to influence lines	304
9.4 Multi-span beams	307
10 Stability of columns	311
10.1 Introduction	311
10.2 The use of potential energy	311
10.3 Euler critical loads	318
10.4 Strut with initial deformation	326
10.5 Struts made from ideal elasto-plastic materials	327
10.6 Double-modulus theory	329
10.7 Tangent-modulus theory	332
10.8 Practical strut formulae	332
10.9 The Southwell method	334
10.10 Energy methods	336
11 Plastic analysis of beams and frames	342
11.1 Introduction	342
11.2 Collapse of redundant beams	343
11.3 Load factor	348
11.4 Basic theorems	349
11.5 Graphical analysis	349
11.6 Virtual-work approach	354
11.7 Combination of mechanisms	358
<i>Suggestions for further reading</i>	368
<i>Answers to problems</i>	369
<i>Index</i>	374

PREFACE

This text has been written primarily for the benefit of undergraduates and I suppose that the preface should be directed towards enlightening the reader about the contents and the reasons for writing a book. I have often wondered if students read a preface; they have in all probability been directed to a particular text by their teachers in the subject, and are far more concerned about the number of worked examples or whether the book appears to be too difficult or too easy for their tastes. However there are a few remarks that I should like to make.

I have been concerned for a number of years with lecturing and demonstrating topics in both structures and elasticity. In addition I have been extremely fortunate in teaching undergraduates in pairs using a tutorial system. This has I hope enabled me to discover the areas of structures that appear to be difficult and that are not sufficiently well covered by existing texts. I am also very conscious of the fundamental errors that are made year after year by successive generations of students. I hope that I shall perhaps eliminate some of these difficulties and also point out the common pitfalls.

The book is intended to serve a student as a basic text in structures. It is not possible to cover all the material required for the various degree courses in a single volume and, by the time the final year of a course is reached, books of a more specialised nature will no doubt be required. The first two chapters are concerned with statically determinate structures; it is still essential to have a firm grasp and feel for this subject, as it is one of the foundations of structures. The other main foundation is elasticity; it would therefore not be possible to make a book on structures self contained without including some elements of elasticity. In three chapters I have attempted to cover all the basic elasticity theory, but I have not dwelt on the more mathematical aspects. An attempt has been made to use a consistent sign-convention throughout the text.

The emphasis in structural teaching has been changing over the past few years, particularly as computers have become freely available. It is however still necessary to have a real understanding of basic structural theory. In chapter 6 the fundamental ideas of stiffness and flexibility are introduced. Also included are examples using work and energy methods in the solution of redundant structures and in the determination of displacements. A separate chapter covers the elements of moment distribution and slope deflection—these are still important methods of analysis that can give a rapid solution if a structure is not too complicated. A

knowledge of matrices is required in chapter 8—it is assumed that all engineers now receive instruction in this subject as part of their mathematics course. This chapter discusses stiffness and flexibility methods, and although these are often covered by a book devoted entirely to the subject, I have attempted to provide sufficient information in one chapter working entirely from the member approach. It may possibly be a little indigestible at a first reading, but the reader will have secured sufficient basic knowledge from the previous chapters and with a little perseverance will win through, particularly as there are several examples worked out in detail using both stiffness and flexibility.

The final three chapters discuss topics that I believe should be covered by a fundamental text. Unfortunately it has not been possible when discussing stability to cover more than the behaviour of a strut in isolation. The last chapter does I hope provide an adequate introduction to plastic collapse so that more definitive works can be read without difficulty.

SI units have been used when any numerical values have been required. Problems for solution are provided at the end of each chapter; a number of these have been taken from Oxford examination papers, by permission of the Clarendon Press, Oxford, but I am responsible for the answers provided.

There can of course be no original material in a book of this nature and I am indebted to the many different sources that have been used over the years in forming my ideas in the presentation of the subject matter.

J. D. TODD

Preface to the second edition

The opportunity has been taken to add extra explanation to improve the clarity of some subject matter. There are some additions to chapter 3 mainly concerned with members that are loaded such that the yield stress is exceeded. The slope-deflection equations in chapter 7 have been written in matrix notation. One of the main changes is in the first part of chapter 8; it was found that the approach used in the first edition was a little difficult for a student meeting the applications of stiffness and flexibility for the first time. The new approach is very much simpler, starting with the derivation of the stiffness matrix for a helical spring. In chapter 10 stability is now introduced from a potential energy approach, this should enable the reader to gain a much better understanding and feel for the subject.