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BASIC MATRICES

An Introduction to Matrix Theory
and Practice

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To Joan

Logical Structure of the Book
This chart shows the principal prerequisites for each chapter

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Preface

There has to be a very good reason indeed for writing yet another book on matrices. Possible justifications could include extreme clarity of exposition making the book a valuable teaching tool, and slanting the material towards a group of people for whom a book on matrices has yet to be written. My own justification lies in the belief that even this well-worked subject may be developed in an original way that is both coherent and useful. It is, of course, of paramount importance that any unorthodox treatment should be useful; originality for its own sake is mere self-indulgence. I have found that the approach that I have adopted has helped me to understand the subject, and I have written it down in the hope that it might help others.

The book is addressed not so much to the mathematician (though I hope that even he may find parts of it useful) as to the engineer, physicist or social scientist. In short, it is addressed to all those who have problems that may be formulated in terms of matrices, and who wish to determine, either by analytical manipulation or numerical solution, the salient features of these problems. Since most problems formulated in terms of matrices find their resolution in a computer print-out, considerable stress is placed on the errors inherent in numerical solution, and to this end much emphasis is placed on norms and condition numbers. The algorithms described may not be the very latest for solving a particular problem, but they are all stable and robust, and may be used in the confidence that they will not, for reasonable problems, give unreasonable solutions. Although detailed error-analyses are beyond the scope of the book, outlines of the basic results are provided.

The approach to the subject is based on an assessment of the needs of those for whom the book is written. It is thus about matrix algebra rather than linear algebra, and with an eye on modern applications it does not seek to emphasise square matrices at the expense of rectangular ones. In keeping with this philosophy the determinant is not used in the development of the theory, its principal properties being outlined in the appendix. The subject proceeds very well without determinants, and the existence of matrix inverses and eigenvalues can easily be established by other means. The other major unorthodoxies of the book occur in the treatment of linear programming. Here the cost is regarded

throughout as a basic variable, and the addition of a 'dummy variable' when dealing with duality introduces a useful symmetry.

My thanks are due to many people, too numerous to name, who have helped both directly and indirectly in the production of this book. I am grateful to Jorge Moré, whose enthusiasm resulted in the decision to go ahead in the first place, and to Bill Hart, who read the manuscript and made many suggestions. Bill's wide mathematical knowledge combined with his ability to see the book from the student's angle resulted in substantial improvements in all departments, and if the book has any value as a teaching text much of the credit must go to him. I am also grateful to Mrs Janet Read and Mrs Pam Short for typing the original manuscript. Pam in particular was at the keyboard when revisions were being made and deadlines passed, and her patience and quiet dedication were an object lesson to me. I am also indebted to my students who acted as unwitting guinea-pigs for my ideas, and who bore the brunt of the various successive approximations to the solution. My dealings, through Malcolm Stewart, with The Macmillan Press have always been extremely amiable and it is a pleasure to record this here. Finally, I am grateful to my wife Joan and my three children, who have tolerated for too long a preoccupied part-time husband and father with great good humour and understanding. Without Joan's forbearance the book would never have been finished, and I dedicate it to her with thanks and affection.

Colchester, Essex
December 1974

C. G. BROYDEN