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Applied Electromagnetics

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Preface

Electromagnetic theory has been a basic subject taught for more than a century to physics students but not to the electrical-engineering student. Before the Second World War the engineer was well grounded in circuit theory but was notoriously weak in field theory; by and large he might have heard of Maxwell's equations but he certainly did not use them. Since the Second World War, many factors have greatly changed the engineer's outlook; particularly the astonishing advances in electronics, in communications (particularly microwaves) and more recently in solid-state devices. Consequently, a basic course in electromagnetics and applications has been included in most first-degree courses in electrical and electronic engineering since about 1950.

The many earlier excellent texts available were unsuitable for engineering courses in electromagnetics for two reasons. First, they had been written from the point of view of the physicist, being more concerned with basic principles than with applications. Second, the introduction of SI (rationalised MKS) units meant that these earlier texts needed to be revised. Consequently the new texts in this subject have been in the main written by and for electrical engineers: as examples see the books by Skilling, Cullwick, Carter, Hayt, and Lorrain and Corson. These excellent texts have been found too advanced and too lengthy for the short time allocated to electromagnetism at Nottingham, that is about fifteen lecture hours in the first year and about twenty in the second year. In the final year there are optional subjects which do develop specialist aspects of electromagnetism such as microwaves and radiation.

It has therefore been convenient to develop this short, compact text which covers the basic material of the first two years without attempting to develop specialist needs that are already well provided. Each author has conducted the second-year course in electromagnetics for about five years, so there has been considerable modification in the choice and presentation of subject matter as a result of feedback from the students. Chapter 10, on low-frequency applications, has been based on notes from Professor L. R. Blake, who has given one or two lectures in the second-year course for several sessions.

Each chapter is provided with a set of about ten problems, mostly of a quantitative nature. Some are provided with answers; the others are deliberately left without. A student can thus be assigned a suitable 'mix' by the tutor. The examples are mostly drawn from the degree examinations at Nottingham. Appendix 2 lists the information made available to students during these examinations to avoid unnecessary memorising.

It is hoped that the text will prove acceptable to first-degree engineering students at universities and polytechnics. We shall welcome information about errors and suggestions for future modifications.

Nottingham
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