ENTROPY

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The significance of the concept of entropy
and its applications
in science and technology

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SECOND EDITION

Revised and Enlarged

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PREFACE TO THE FIRST ENGLISH EDITION

The entropy concept, which plays a role as important as that of the energy concept in every field of physics, chemistry and technology, generally presents many difficulties. This was the reason which induced me in 1947 to write a book about entropy in a form clear enough to be read without undue difficulty by the chemist and physicist. This work met with such success that a second edition became inevitable. This edition, in contrast to the first, appears not only in Dutch, but also in English, French, German, Spanish and Polish.

In the first edition I limited myself to discussing the basic principles of thermodynamics, statistical mechanics and quantum mechanics only so far as was necessary to explain the entropy concept and to calculate the entropies of monatomic and diatomic gases by the two methods provided by thermodynamics and statistical mechanics. The success of the first edition encouraged me to increase considerably, in the present edition, the number of examples of the applications of the entropy concept. Some of these examples lie in the domain of chemistry, others in that of the physics and physical chemistry of solids. The subjects discussed include paramagnetic and ferromagnetic substances, interstitial and substitutional solid solutions, internal friction, order-disorder transformations and precipitation in alloys, the structure of glass, the electronic theory of metals and semi-conductors, the structure of martensite, the elasticity of rubber, the specific heats of solids and the existence of vacancies in solids. Furthermore, attention is paid to solutions of polymers, the calculation of chemical equilibrium, the efficiency of heat engines, the attainment of extremely low temperatures and the radiation of heat and light. In contrast to the Dutch and German editions, the English edition contains an additional section on fuel cells and heat pumps (Section 3.14).

It is obviously impossible, and was certainly never my intention, to give a complete survey of all questions relating to the concept of entropy. For this reason the reader will search in vain for the thermodynamics of irreversible processes and the accompanying "production of entropy". Nor will he find anything about the use (and frequent abuse) of the entropy concept in information theory. This limitation and the considerable reduction of the

chapters dedicated to gases have made it possible to limit this book to a reasonable number of pages.

It is probable that some readers will lack the time to study this work thoroughly. For their benefit, I have arranged the subject matter in such a way that the first part of the book (Chapters 1. to 3) forms a complete and coherent unit. They have only to look through this section to realize that the elementary concepts of thermodynamics and statistical mechanics explained in the first two chapters are sufficient to tackle the numerous problems dealt with in Chapter 3.

The principles of quantum mechanics and quantum statistics, which make possible a detailed discussion of the basis of the entropy concept, are not explained until Chapter 4. This chapter is scarcely more complicated or difficult than the preceding ones and the reader will be forced to admit that quantum statistics are logically more satisfactory and, for that reason, that they are "simpler" in many ways than classical statistics. The last two chapters have been devoted to rarefied monatomic and diatomic gases. It is a happy circumstance that the thermodynamic properties of these gases, as well as the chemical equilibria between these gases can be calculated exactly from spectroscopic data, while the calculations carried out for solids in the first chapters are only approximate, since the theory of the solid state is not yet sufficiently developed to permit of accurate calculations.

In accordance with the nature of this book, the treatment is not extremely rigorous. Without sacrificing more precision than necessary, I have endeavoured to reach all of those students and researchers to whom thermodynamics and statistical mechanics seem a little frightening, although a certain knowledge of these subjects is indispensable to them. With this aim in view, I have intentionally avoided mentioning several difficulties of a fundamental nature in discussing statistical mechanics. I am convinced that, for the majority of readers, this voluntary omission offers more advantages than it causes inconvenience and I dare to hope that the theoreticians will not take offence. Moreover, these principles have been set out in detail in several classic works (for example R. C. Tolman, *The Principles of Statistical Mechanics*, Oxford, 1938).

Several friends and colleagues have been so kind as to read part or all of the manuscript before going to press. I would like to take this opportunity to express my appreciation. Their criticisms, and in particular those of Professors J. L. Meijering, D. Polder and J. H. van Santen, and Drs. W. de Groot, H. J. G. Meyer and J. Smit, have enabled me to introduce several improvements in this book. I also wish to thank Mrs. Mulder-Woolcock, B.Sc., for translating the Dutch text into English and Mr. C. Hargreaves,

M.Sc., for checking the text as regards technical terms, as well as Mr. M. B. Verrijp for drawing the diagrams and correcting the proofs.

Eindhoven, March 1962

J. D. FAST

PREFACE TO THE SECOND EDITION

The greater part of the original printing has been retained. In addition to the correction of several errors and the introduction of a few minor improvements, the section dealing with *fuel cells* has been extended. Some new material has been added in an Appendix. The new topics include a discussion of "negative absolute temperatures" and *lasers*, and a treatment of the thermodynamics of *superconductors*.

Eindhoven, March 1968

J. D. FAST

NOTE TO 1970 REPRINT

This reprint contains an appendix explaining my omission from the book of a discussion of the entropy concept in information theory and communications engineering. I believe that the term entropy is misused in this context; information and entropy have different dimensions and the relationship between the two is only of a formal mathematical nature.

The reader is asked to add a negative sign to the right side of Equation (1.6.7) on page 16, in order to reconcile it with the convention concerning the sign of the electromotive force of a cell. The latter is called positive if the chemical reaction involved is so written that it proceeds spontaneously from left to right when the cell is short-circuited.

Eindhoven, April 1970

J. D. FAST

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