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Basic Solid Mechanics

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Preface

This book has been compiled from lecture notes and examples that I have used in my teaching of solid mechanics, strength of materials and structures over many years. It is intended for undergraduate engineering courses in which statics, solid mechanics and structures are taught to an intermediary level. The contents should serve most courses in mechanical, civil, aeronautical and materials engineering. The approach employed is to intersperse theory with many illustrative examples and exercises. As readers work through these they should soon realise the engineer's interest in solid mechanics. They will see that all calculations made are related to the load carrying capacity and the deformation of materials used in engineering design. Among the design considerations are: the choice of material, its physical shape, the nature of imposed loading and its effect on the internal stress and strain. The loadings refer to: tension, compression, bending, torsion and shear. Typical structures to which these loadings are applied include: bars, columns, struts, tubes, vessels, beams, springs and frames.

The analyses of stress, strain, load and deflection employ various techniques with which the reader should soon become familiar. Among these are: Mohr's circle, the free-body diagram, Hooke's law, Macaulay's method and Castigliano's theorems. The text illustrates where and how best to employ each technique within a logical presentation of the subject matter.

In general, a unique solution to a problem in solid mechanics will satisfy equilibrium, compatibility and the boundary conditions. Throughout this book the three conditions have been satisfied to provide closed solutions. However, it may not always be possible to achieve this as the loading and geometry become more complex. The final chapter shows how the known stiffness matrix for three different finite elements can be embodied within a numerical solution to displacement, stress and strain. The three aforementioned conditions are satisfied but, because it is necessary to assume a displacement function, the solutions found are close approximations. Finite elements can offer no advantage where closed solutions exist.

An overview of the contents is given in Chapter 1. There the reader will also find a tribute to those early engineers and men of science who are responsible for shaping this subject into the form in which it is taught

today. They have provided us with the techniques and a system of units with which to define the subject.

Exercise sections have been compiled by the author to support the topics within each chapter. They have been derived mostly from past examination papers. Acknowledgements are due to Brunel University, Kingston University and to the Council of Engineering Institutions for granting permission to include questions from their past examination papers both as worked examples and exercises.

D.W.A. Rees

List of Symbols

a, f, A, B, C, R, V	force vectors
E, E'	strain matrices
K	stiffness matrix
L	rotation matrix
M, T	matrices of direction cosines
S, S'	stress matrices
d	displacement vector
a, l, L	lengths
A	section area, enclosed area
b, d, e, s	dimensions
B, C, F, R, p	forces
d, D	bar diameters
E, G, K	elastic constants
F	shear force
F_{ij}	flexibility components
h, k	force locations
i, D	first moment of area
I	second moment of area
J	polar moment of area
J_1', J_2', J_3'	stress deviator invariants
k	radius of gyration, shear yield stress
K_{ij}	stiffness coefficients
M	bending moment
n	hardening exponent
N	rotational speed
p	pressure, pitch
P	applied force, power
q	shear flow
Q	heat work transfer
r, θ, z	polar co-ordinates
R	radius of curvature
S	safety factor
t	thickness, temperature
T	torque
u	radial displacement, strain energy density
U	strain energy stored
v	lateral strut deflection

V	volume
w	distributed loading, warping displacement
W	work transfer, applied load
x	extension
x, y, z	cartesian co-ordinates
$\bar{x}, \bar{y}, \bar{z}$	centroid locations
y	distance from neutral axis
Y	tensile yield stress
α	coefficient of expansion, helix angle
β, θ	inclinations
δ, Δ	load point displacements
ε	direct strain
γ	shear strain
ϕ	shear distortion, rotation
ν	Poisson's ratio
σ	direct stress
σ_m	mean stress
σ_c	compressive yield stress
σ'	deviatoric stress
τ	shear stress

Subscripts and superscripts

c	compression
e	element
E	Euler, equivalent, external
i, j	tensor notation, components of a matrix
I	internal
k	components of a vector quantity
m	mean or hydrostatic
o	octahedral
r, θ	polar components
s	shear component, solid
T	transpose of a matrix
v	volumetric component
x, y	cartesian components
$'$	co-ordinate rotation, deviatoric
$*$	complementary measure
1, 2	principal values

Abbreviations

e.t.b.	engineer's theory of bending
e.t.t.	engineer's theory of torsion
NA	neutral axis
PS	proof stress
PVW	principle of virtual work
SPE	stationary potential energy
UBS	ultimate bearing strength
UCS	ultimate compressive strength
UTS	ultimate tensile strength