Good design for effective revision

Full coverage of the subjects included in the examinations

The methods, techniques and concepts you need to know, and examples illustrating them

Real exam questions of different types:
– give you practice on the real thing
– can be set as tests or self-tests

Answers at the end of the book help you check your progress

Worked examples show you the answers the examiners expect

Glossary help you with difficult terms

Good design

for effective revision

2 Collecting Information: Surveys

Objectives By the end of this chapter the following terms and methods should be understood:

	Date attempted	Date completed	Self-assessment
Survey design			
Survey methods			
Observation			
Interviewing			
Questionnaires			
Sampling			
Sampling methods			
Random sampling			
Quota sampling			
Panels			

Summary This chapter describes survey methods used for collecting statistical information. It explains statistical sampling techniques used in surveys.

Start and completion column Keeps tabs on your progress – see at a glance which areas still need to be worked through

Self-assessment For you to note how well you've done or areas which need to be revised again

Chapter breakdown Shows you in detail what is covered in the chapters

Introduction

How to use this book

The aim of this book is to provide a full, thorough and concise coverage of the subjects included in the GCSE Statistics examinations, and the statistics in a range of business, professional and management courses, diploma and degree examinations.

All statistics examinations are different in their requirements, one concentrating on one area of techniques and methods, and another on other areas. This book covers all those examinations and courses that do not require advanced mathematical knowledge but are concerned with basic mathematical and statistical methods and their applications.

Each chapter is structured so that a description of techniques is followed by examples, questions and assignments with solutions and worked examples plus large numbers of recent examination questions. In order to make the best use of this book, you should study the techniques carefully, work through the examples and then attempt the questions and exercises so that you have plenty of practice in each section and gain experience in formulating answers.

There is a balance in each chapter between those questions for which solutions have been provided in the text and those for which the numerical answers are at the end of the book. In Chapter 6, for example, 48 questions have solutions provided in the text while 26 have answers at the end of the book. All the questions are examination questions or are based very closely on them, whether they are classified as worked examples, questions or exercises. Questions with answers at the end of the book can be set as tests or self-tests and the answers can be computed by following the methods set out in the chapter.

Examination papers have been analysed so that the book includes a wide spread of statistical methods, techniques and concepts to provide a solid foundation for success in understanding statistics for a broad range of people

some of whom may have very limited mathematical knowledge. The book includes a section on basic mathematics as a reminder of simple numerical techniques.

This book should be read in conjunction with *Mastering Statistics* (Tim Hannagan, Macmillan) which is complementary in its scope with a more detailed description and analysis of statistical techniques and concepts.

Statistics in practice

Statistics is a practical subject concerned with obtaining and processing data in order to extract numerical information and to make inferences which will help in making decisions. GCSE syllabuses and vocational courses encourage the use of statistics in practice in order to facilitate learning. Practical work in statistics can develop the understanding of the subject and the application of statistical methods.

Practical work can develop skills in:

- identifying the statistical aspects of a problem;
- collecting data;
- selecting appropriate statistical methods;
- interpreting statistical evidence;
- communicating statistical results.

Practical work in statistics can include:

Measuring – (length, weight, volume/capacity, time, temperature);

Counting – (heads/tails, dice, vehicles, people);

Surveying – (opinions, results).

Course work on statistical techniques can be based on this practical work. Statistical comparisons can be made on samples of products and relative prices, weight or numbers; correlation can be based on age and weight, height and weight, price and sales; time series can be based on wealth, prices, population.

The importance of reliable data means that it is essential to have an understanding of the difficulties of producing accurate results from questionnaires and experiments and in the use of sampling and there needs to be an awareness of the problems of using published statistics (secondary sources) and the importance of defining terms very carefully. The construction of tables and diagrams can help in the understanding of their appropriateness and the reliability of published diagrams. Sources of data include newspapers and magazines, television programmes, pamphlets and government publications.

The practical application of statistics can be an important element in understanding the subject, while forming the basis for course work and support for the work required in order to be successful in examinations.

How to revise

An examination involves answering questions over a few hours on what has been learnt over a year or two and is a test of the ability to extract the central points from the syllabus. However much work has been carried out during the time leading up to an examination, a period of intensive revision will be necessary. Revision should start at least six to eight weeks before the examinations and should be based on a timetable so that each topic is revised in turn with more time concentrated on 'weak' areas.

It is useful to have built up a series of notes covering the whole syllabus to provide a basis for revision. The content of these notes should be determined by individual needs for reminders of statistical methods and applications. Headings and sections, lists of formulae and methods all help in 'last minute' revision.

Examinations are based on knowledge and memory as well as understanding. In statistics this means that techniques need to be followed, formulae and definitions remembered and applications and examples prepared so that they can be used in a relevant way to illustrate and explain concepts.

Above all in preparing for a statistics examination, practice is essential, particularly of calculations and preferably under examination conditions.

How to plan for a statistics exam in 3 steps

In planning for a statistics examination it is essential to obtain certain things and to practise calculations, as follows:

Step 1 Obtain the syllabus for the examination

The syllabus will show what will be examined and how it will be examined. It is important to know the type of questions which will be asked, whether there is a choice and how many questions have to be answered. In most statistics examinations calculators can be used and papers are set assuming their use. This means that questions may include a number of calculations which would take a long time without a calculator but can be completed quickly with one.

Statistics exams seldom involve long essay questions but tend to include a variety of other types of questions, such as:

- *calculation questions*, where a calculation or a number of calculations are asked for;
- *problem-solving questions*, where a problem is posed which might involve in its solution a combination of calculations, analysis, comment and discussion;
- *interpretation questions*, where an analysis is asked for of a table, diagram or graph or of a solution to a calculation;
- *multiple-choice* or '*objective*' *test questions*, where several alternative answers are given and the correct one has to be ticked;
- *short answer questions*, where a two line written answer is required to what is often an interpretation question.

There is a growing tendency among examination bodies to ask a variety of types of question. Multiple-choice and short answer questions are asked in order to test the whole syllabus, while longer problem-solving questions test particular parts of the syllabus in more detail.

Step 2 Obtain the marking schemes and examiners' reports for the examination

In answering examination papers it is essential to know how marks are distributed over the questions and as far as possible for each part of a question, because this distribution will determine the length of time spent on each section of the paper.

The instructions on the examination paper will invariably indicate the distribution of marks between the questions, sometimes by stating that 'all questions carry equal marks'. In the case of some examinations the marks for each part of a question are shown on the paper, other examining boards include marking schemes in examiners' reports.

In short calculation questions most, if not all, of the marks are for a correct answer; in problem-solving questions marks are given for the various sections of the question and marks may be given for correct methods or consistent answers even when the solution is not entirely accurate; while interpretation questions will be marked on the basis of the width and depth of the comments, discussion and analysis. Multiple-choice questions are usually marked right or wrong, a mark being awarded for a correct answer. It is very, very rare for marks to be deducted for a wrong answer, or for any marks to be awarded for a 'second best' answer, and therefore it is sensible to answer all multiple-choice questions even if the answer is a guess.

Step 3 Practice

Examiners' reports point out that an examination in statistics always involves candidates in calculating some of the statistical measures which are specifically mentioned in the syllabus. It is very important to practise these calculations so that the methods are fully understood.

It is important also, to have an idea of what the answer should be to any calculation. This means that by looking at a set of figures it should be possible, with practice, to have a good idea of the solution, that is whether it will be around say one, ten, a hundred or a thousand. This will make it possible to notice immediately if a large error is made in a calculation which leads to an 'impossible' answer.

Which questions to expect in your statistics exam

In preparing for an examination, an obvious query is 'what are the most frequently asked questions?' Trying to predict or 'spot' questions in an examination is dangerous; it is fairly safe, however, to predict topics on the basis that these are the central areas of the syllabus. Past papers show which subjects appear most frequently:

(a) GCSE examinations

The aims of GCSE syllabuses are to produce courses which will enable the achievement of the acquisition of statistical knowledge and skills; the development of an understanding of statistical methods and concepts and awareness of their power and limitations; the development of an awareness of the breadth of application of statistics and of an ability to apply and interpret statistics in everyday situations and in other disciplines.

The examinations assess a candidate's ability to recognise, recall and use statistical language and facts, perform relevant computations to appropriate degrees of accuracy, select, present and analyse data, reason logically and make statistical inferences. There are three types of paper: an objective test paper, short answer paper and a structured question paper. The GCSE boards favour questions on:

- frequency distributions
- classification
- presentation of data through diagrams and graphs
- averages, particularly the mean and median
- dispersion, particularly the standard deviation and interquartile range
- correlation, both product moment and rank coefficients
- probability

Questions may also be asked on

- quantitative/qualitative information
- survey questions/sampling
- maximum and minimum values

- percentages and proportions
- death rates
- the geometric mean
- z-values
- index numbers
- moving averages and semi-averages

Questions are less likely to be asked on a description of statistics, or in detail on survey methods, financial mathematics, Z-charts, Lorenz curves, significance tests and confidence limits, seasonal variations and the least squares method of calculating a linear trend.

(b) Business, professional and management examinations

- (i) RSA papers include more questions than in the GCSE on survey methods and sampling, Z-charts and Lorenz curves, index numbers, correlation and trends. RSA papers include fewer straightforward questions on averages, dispersion and probability.
- (ii) The Chartered Institute of Marketing papers have many questions on surveys and sampling as would be expected and also many questions on index numbers, correlation and trends.
- (iii) Statistics papers set by the accountancy bodies and the CIMA include questions on financial mathematics, presentation, index numbers, correlation and trends. These examination papers tend to consist of long 'problem solving' questions.
- (iv) GNVQ and BTEC papers are in the main set locally by colleges and therefore papers vary from centre to centre. They often include questions on financial statistics, significance tests, confidence limits, index numbers and trends as well as the areas covered in (a) above.
- (v) All of these papers in (b) are more concerned with the application of statistics than the GCSE papers and in the case of (ii), (iii) and (iv) the application of statistics to particular areas of business.

Remember in the exam

- Point 1: *to answer the question that is asked* and not the question that it is hoped would be asked.
- Point 2: to read the instructions carefully to be sure how many questions have to be answered and which (if any) are compulsory.
- Point 3: to make a timetable for the examination so that the time available is used to the best advantage (e.g. if five questions carrying equal marks are to be answered over three hours, thirty-five minutes could be allowed for each question, leaving five minutes for planning and checking).
- Point 4: to answer all the required questions; it is particularly easy in questions requiring calculations to become stuck on the detail of the mathematics and waste time which would be better spent on the next question (i.e. it is easier to score nine out of twenty marks on each of two answers, than eighteen out of twenty on one).
- Point 5: to make plans for questions where necessary; statistics questions are often well structured and planning can be minimal, but plans can be used for deciding which questions to answer and which to leave when there is a choice, and insuring a good and logical coverage of points in questions that require comment and discussion.
- Point 6: to decide exactly what the examiner wants in the way of calculations, facts, figures, diagrams and comments. It helps to look for key words such as: describe, outline, analyse, calculate and so on, and to follow the instructions so that *the question actually asked is fully answered*.

Exam Board Addresses

For syllabuses and past papers contact the Publications Office at the following addresses:

Midland Examining Group (MEG)
c/o University of Cambridge Local Examinations
Syndicate
1 Hills Road
CAMBRIDGE
CB1 2EU
Tel. 01223 553311

Southern Examining Group (SEG)
Publications Department
Stag Hill House
GUILDFORD
Surrey
GU2 5XJ
Tel. 01483 302302 (Direct line)

Northern Examinations and Assessment Board (NEAB)
12 Harter Street
MANCHESTER
M1 6HL
Tel. 0161 953 1170
(Also shop at the above address)

University of London Examinations and Assessment Council (ULEAC)
Stewart House
32 Russell Square
LONDON
WC1B 5DN
Tel. 0171 331 4000

Northern Ireland Council for the Curriculum, Examinations and Assessment (NICCEA)
Beechill House
42 Beechill Road
BELFAST
BT8 4RS
Tel. 01232 704666

Welsh Joint Education Committee (WJEC)
245 Western Avenue
Llandaff
CARDIFF
CF5 2YX

Scottish Examination Board (SEB)
For full syllabuses
Ironmills Road
Dalkeith
MIDLOTHIAN
EH22 1LE
Tel. 0131-663 6601

Or recent papers from the SEB's agent
Robert Gibson & Sons Ltd
17 Fitzroy Place
GLASGOW
G3 7SF
Tel. 0141-248 5674

Remember to check your syllabus number with your teacher!

Addresses of Professional Examining Boards

Syllabuses and past papers for examinations administered by business, professional and management bodies can be obtained from the following addresses:

Association of Certified Accounts (CACA)
Syllabus from:
29 Lincoln's Inn Fields
LONDON WC2A 3EE
Tel: 0171 242 6855
Fax: 0171 396 5757
Past questions and papers from:
Student Publication Dept
1 Woodside Place
GLASGOW G3 7QS
Tel: 0141 309 4040
Fax: 0141 309 4041

Business and Technology Education Council
Central House
Upper Woburn Place
LONDON WC1H 0HH
Tel: 0171 413 8400
Fax: 0171 387 6068

Institute of Chartered Accountants
Gloucester House
399 Filbury Boulevard
CENTRAL MILTON KEYNES
Bucks MK9 2HL
Tel: 01908 668833
Fax: 01908 691165

Institute of Chartered Secretaries and Administrators (ICSA)
16 Park Crescent
LONDON W1N 4AH
Tel: 0171 580 4741
Fax: 0171 323 1132

Chartered Institute of Management Accountants (CIMA)
63 Portland Place
LONDON W1
Tel: 0171 637 2311
Fax: 0171 631 5309

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Moor Hall
COOKHAM
Berks SL6 9QH
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Fax: 01628 531382

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IPD House
35 Camp Road
LONDON SW19 4UX
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Westwood Way
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To Gladys and Grace

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