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Plastic Methods for Steel and Concrete Structures

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Second Edition

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Preface

In the preface to the first edition I said that this book owes much to the many students, both undergraduate and postgraduate, who have sat patiently and attentively through my lectures on plastic methods. This continues to be the case with the second edition. Questions and discussion during and outside the lectures have shown that certain aspects of the subject consistently create difficulties and have also pointed to where my explanations in the first edition were not clear. Perhaps I flatter myself that the difficulties in the subject are not due to shortcomings in my teaching technique, but because certain points need greater attention than is possible in a lecture. It has been my intention to provide that extra attention in this book.

The second edition was harder to write than the first – it has become difficult to find and justify the time required. However, there were several motivations that made it possible. Firstly, I had identified various shortcomings in the original which was also rather dated in places. Secondly, things have moved on in the design of steel and concrete structures. New British, European and US design codes have been introduced in the last ten years which are explicit in how plastic methods should be used. It really was essential to include some of that material. Finally, I came under pressure from my publisher!

The book is intended to cover most of the requirements of students at both undergraduate and postgraduate levels. It will also be useful to practising engineers who use plastic methods. The emphasis throughout is on the ideas behind, and the application of, plastic methods, rather than their mathematical justification (I have given references to appropriate texts and papers for this). To achieve my aim I have simplified some of the arguments (as was pointed out in a conference paper shortly after the publication of the first edition). I am sorry if my approach offends the purists.

The book is divided into eight chapters. The first two deal with the concepts of plastic behaviour, and plastic bending in particular. Chapters 3 and 4 describe the techniques for finding the collapse loads of steel frames. Chapter 5 looks at the effects of deflections, imperfections and instability on the collapse of structures. These ‘secondary’ effects are complex and a detailed study is be-

yond the scope of the book. However, I have endeavoured to give a general explanation and have then shown how these effects are covered in design. Chapter 6 shows how plastic methods are used in the design of steel structures, looking in detail at beams and portal frames. Chapters 7 and 8 deal with reinforced concrete structures: Chapter 7 examines the problems of applying plastic methods to concrete frames and the compromises that have to be made in design codes, and Chapter 8 describes the powerful yield-line and strip methods for slabs. Since the publication of the first edition, computers have made an enormous impact on technology. This edition shows how computing can be applied to the use of plastic methods.

As a rough guide based on my own courses, the first three chapters provide a solid introduction which would be appropriate to second year undergraduate level, while Chapters 4, 6 and parts of 5, 7 and 8 would be suitable for a third or fourth year course. A postgraduate course could make use of all the material.

I have tried to be consistent in the layout of each chapter. There is an introduction which gives, where appropriate, the background theory. The meat of each chapter is usually presented in a series of examples which have been carefully graded. The first example introduces the ideas in the simplest possible manner, while subsequent ones introduce new ideas or examine those areas that can cause difficulties. *It is important that the reader realises that the examples introduce new ideas as well as illustrate earlier ones.* The whole is then brought together in a summary. At the end of each chapter (apart from the first) there is a series of examples, designed to bring out the various points made in the chapter. I strongly encourage the reader to work through the examples, preferably in order. Practice really is the best way of getting to grips with plastic methods.

Stuart S. J. Moy

Notation

It has been convenient for the same symbols to have different meanings or for similar things to have different symbols. These are all noted here, but are also defined in the text. It has been necessary to add suffixes to various symbols to indicate specific meanings – these are always defined in the text.

Forces

C, T	compressive and tensile forces from bending stress blocks
F	axial member force in a pin-jointed truss
N	shear force in a member in bending
P	axial force in a member in bending
P_c	axial force at collapse
P_{cr}	buckling force
P_p	squash load
q	distributed loading on a slab
SF	shear force (abbreviation)
UDL	uniformly distributed load (abbreviation)
V, H	vertical and horizontal applied loads (forces) and reactions
W	applied load
w, Q	line load
λ	load factor
λ'	change in load factor
λ_c	collapse load factor
λ_e	buckling load factor
λ_l	lower bound (under-estimate) to the collapse load factor
λ_u	upper bound (over-estimate) to the collapse load factor

Moments

BM	bending moment (abbreviation)
BMD	bending moment diagram (abbreviation)

FEM	fixed-end moment (abbreviation)
M	moment; magnitude of a bending moment; moment of resistance per unit length (slabs)
M_e	bending moment found by plastic analysis
M_n	moment per unit length about an axis along a yield line
M_p	plastic moment
M'_p	reduced plastic moment
β_b	redistribution ratio (BS 8110 notation)
δ	redistribution factor (Eurocode 2 notation)

Material and Section Properties

A	cross-sectional area
A_s	cross-sectional area of steel reinforcement
B, b	width
D, d	depth
d	effective depth of tensile reinforcement
d_1	effective depth of compression reinforcement
E	Young's (elastic) modulus
E_{sh}	slope of stress-strain curve at the start of strain hardening
f_{ck}	characteristic cylinder strength of concrete
f_{cu}	characteristic cube strength of concrete
G	structure weight function
G'	variable part of weight function
g	weight per unit length
I	second moment of area
l	effective length
l/r	slenderness ratio
r	radius of gyration [$= \sqrt{(I/A)}$]
S	plastic modulus
T	flange thickness
t	web thickness, thickness
x	depth to neutral axis (axis of zero strain) in a concrete beam
Z	elastic (section) modulus
γ	shear strain
ϵ	direct strain
σ	direct stress
σ_y	yield stress
τ	shear stress
τ_y	yield shear stress

Lengths and Displacements

e	eccentricity
L, h	overall dimensions

l, a, b	dimensions
R	radius, radius of curvature
x, y, z	coordinate axes; distances in coordinate directions
$\alpha, \beta, \gamma, \theta, \phi$	angles; rotations
Δ, δ	displacements, deflections
χ	curvature

General

c, n, k	constants, ratios
i	ratio of positive and negative moments of resistance
K	Hillerborg correction factor
m	number of elementary mechanisms
n	number of plastic hinges
p	number of possible plastic hinges
r	degree of redundancy (indeterminacy)
s, c, m	stability functions
YL	yield line (abbreviation)
μ	measure of orthotropy in slabs

Sign Convention

The most convenient sign conventions have been used. Thus tensile stresses, strains and axial forces are assumed positive except in Chapter 5 where compression is assumed to be positive. Bending moments are plotted on the side of the member that is in tension. Tension on the underside of a beam or to the left of a column, as drawn, is assumed to be caused by a positive bending moment.