Mastering

Advanced Pure Mathematics

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Advanced Pure Mathematics

Geoff Buckwell

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Introduction

Mastering Advanced Pure Mathematics is not just another A-level text book. It contains full explanations so that the book can be used at home as a support in the study of a wide range of syllabuses based on the common core A-level topic list. Throughout the book, there are a number of boxes headed DO YOU KNOW? These contain brief reminders of ideas. If you are happy with the ideas covered in DO YOU KNOW?, then you should be able to continue without any problems. Any ideas in the DO YOU KNOW sections you are not sure about can be studied in *Mastering Mathematics* (also in this series). So often, people try to study topics without the necessary background knowledge. *Mastering Advanced Pure Mathematics* helps you to avoid making that sort of mistake.

You will also notice boxes throughout the book headed MEMORY JOGGER. These points are extremely important. Try to remember them, as they will increase your level of understanding. In order to check that you have understood the work, you can try the exercises: these have been arranged so that you can check your progress as you go along. There are also miscellaneous examples at the end of each chapter that you should be able to complete on first reading. The revision problems at the ends of the chapters are more difficult and should not be attempted until you are familiar with most of the book, because often, questions in a chapter will contain topics covered in other chapters.

Scattered throughout the book are activities designed to broaden your knowledge. It is hoped that these will help your enjoyment of mathematics.

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