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Numerical Analysis

Peter R. Turner

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***To Pat for her patience,
and to my great friends and mentors
Charles Clenshaw and Frank Olver***

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Preface

What makes this book different from all the other books on Numerical Analysis and Scientific Computing? The most important difference is that this book was *not* written as a standard text for a course but rather as a student study guide to accompany such a course. That is not to say that it could not be used as a primary course text – indeed I have used it (successfully) in that capacity when the text for a course was unavailable at the appropriate time.

As a study guide, material is not presented in conventional ‘theorem–proof’ format but rather as a succession of examples and exercises which vary both in length and difficulty. Each chapter opens with a Topic Guide which forms an extended contents list for the chapter and each section begins with a (usually) brief summary of its basic theory and methods. Sections are for the most part quite short to ease navigation through the various topics.

What about the level and choice of material that is covered? This book does not attempt to encapsulate all of the current thinking and most up-to-date algorithms for every numerical problem in just a couple of hundred pages. The coverage is intended to include all the topics which might be encountered in a one term, semester or even one year introductory course in numerical methods, numerical analysis or scientific computing. Not all of the topics are likely to be included in all such courses but I anticipate that at least 95% of the material of any such course is covered. Because of the nature of the courses for which it may be a useful companion, this book does not have advanced prerequisites. In particular, only a rudimentary knowledge of linear algebra and multivariate calculus is assumed.

Most introductory numerical courses include some computer programming. Often this is in a specified language, sometimes not; often they use packages to avoid programming from scratch. Most of the important methods discussed here are presented either in detailed algorithmic form, which should be fairly readily implemented in whatever language or package is being used, or as Turbo Pascal programs. The latter are mostly ‘stand-alone’ one-off programs (although sometimes reference is made to linear algebra programs) which are intended to be easy to read and therefore easily translated into any other structured programming language. At times this militates against using the most polished programming style – the intention is to provide (or at least ease) understanding, not a commercial software package!

There will, I hope, be many points in this book where you will wish for greater depth of coverage and will want to know more of some aspect of the methods under discussion – perhaps more analysis, perhaps more on the practical aspects or on more advanced topics. If so then your introductory course and this book have succeeded; I recommend two things: one is to choose to study a more advanced numerical analysis course at the first opportunity and the other is to consult one of the many more extensive texts that are available. (I do not give any specific recommendations since I admit to a certain partiality!)

Finally, I must acknowledge the help and assistance of several friends and colleagues who have offered advice (sometimes taken) and encouragement. Most especially, I owe a debt of gratitude to Dennis Frazier who not only read the drafts carefully but even worked most of the examples himself and so saved me from some embarrassing errors! Those that remain are mine.

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Annapolis, Maryland, USA
1994