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MACMILLAN COLLEGE WORK OUT SERIES

Mechanics

Phil Dyke

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10	9	8	7	6	5	4	3	2	1
04	03	02	01	00	99	98	97	96	95

To my son Adrian

Contents

Preface	ix
Acknowledgements	xi
1 Revision of Preliminary Ideas	
1.1 Fact Sheet	1
1.2 Worked Examples	2
1.3 Exercises	8
1.4 Outline Solutions to Exercises	8
2 Statics	
2.1 Fact Sheet	10
2.2 Worked Examples	11
2.3 Exercises	21
2.4 Outline Solutions to Exercises	22
3 Motion Under Gravity	
3.1 Fact Sheet	25
3.2 Worked Examples	25
3.3 Exercises	39
3.4 Outline Solutions to Exercises	39
4 Linear Momentum	
4.1 Fact Sheet	42
4.2 Worked Examples	43
4.3 Exercises	54
4.4 Outline Solutions to Exercises	55
5 Variable Mass	
5.1 Fact Sheet	58
5.2 Worked Examples	58
5.3 Exercises	66
5.4 Outline Solutions to Exercises	67
6 Vibrations	
6.1 Fact Sheet	70
6.2 Worked Examples	71
6.3 Exercises	89
6.4 Outline Solutions to Exercises	90
7 Circular Motion and Rotating Axes	
7.1 Fact Sheet	93
7.2 Worked Examples	94

7.3	Exercises	106
7.4	Outline Solutions to Exercises	107
8	Orbits	
8.1	Fact Sheet	110
8.2	Worked Examples	111
8.3	Exercises	123
8.4	Outline Solutions to Exercises	124
9	Rigid Bodies	
9.1	Fact Sheet	127
9.2	Worked Examples	128
9.3	Exercises	145
9.4	Outline Solutions to Exercises	146
10	Energy, Impulse and Stability	
10.1	Fact Sheet	150
10.2	Worked Examples	151
10.3	Exercises	163
10.4	Outline Solutions to Exercises	164
11	Lagrangian Dynamics	
11.1	Fact Sheet	168
11.2	Worked Examples	169
11.3	Exercises	186
11.4	Outline Solutions to Exercises	187
12	Non-linear Dynamics	
12.1	Fact Sheet	192
12.2	Worked Examples	193
12.3	Exercises	202
12.4	Outline Solutions to Exercises	203
Appendix A: Vector Calculus		206
Appendix B: Differential Equations		209
Bibliography		212
Index		213

Preface

Since the time of Isaac Newton (1642–1727) and probably before, mechanics has played a central role in mathematics and physics. It is also essential for other subjects such as physical chemistry and civil, mechanical and aeronautical engineering.

Mathematics, and those subjects that use mathematics extensively, such as physics and engineering, are based on skill. In much the same way as sportsmen and sportswomen learn by doing, so do mathematicians and allied professionals learn by solving problems. Initially of course the golfer needs to know how to grip the club and how to stand, the pianist needs to know the rudiments of the keyboard and musical notation. Similarly, the student of mechanics needs to know the tools of the trade, in this case some algebra and calculus, a little geometry and trigonometry and some familiarity with vectors. After this, however, there is no substitute for doing problems. Many students have great difficulty getting started on a problem. One of the merits of this *Work Out* series is that the student learns precisely how to start by watching someone else (in this case me) doing problems. As I do the problems, I talk the reader through my thinking processes, so there should be no doubt why particular steps are being taken and why we are doing what we are doing at any particular stage. It has been said that of all skills-based subjects, mechanics stands out as being unique in benefitting students through exposing them to solved problems. I agree wholeheartedly with this. I learnt most of my mechanics by seeing others solve problems. The ones I have selected for this book include all my favourites. (I used to get so annoyed when the books I consulted never seemed to solve the hard problems. I sincerely hope that this criticism cannot be levelled here.) There are, at the end of each chapter, some exercises you should try for yourself. Outline solutions to all exercises are available at the very end of each set of exercises, but please resist the temptation to cheat!

Before running through the contents, some comments about student background and computer algebra are necessary. In the last ten years or so, the amount of algebraic manipulation required of schoolchildren in pre-16 mathematics has dramatically decreased. This has occurred in an environment when more assessment is in the form of course work rather than examination. It is much more difficult to assess skills such as algebraic manipulation (or spelling for that matter) outside the formal examination. In my view, the decrease in algebra skills has happened by default rather than by design. Parallel with this has been the loss of geometry at school level. Both of these have changed the kind of mechanics problems that can be tackled by the average student. I have taken this into account in selecting the problems. This brings me to computer algebra. I have, a little reluctantly, decided not to emphasise computer algebra. There are many packages available (Macsyma[®], Derive[®], Maple[®] and Mathematica[®] to name but a few). There are also many other excellent software packages, which will increasingly affect the education of mathematicians. (I have used Omnigraph[®] extensively in Chapter 12 for example). As yet there is no standard, and to base a problem-solving text such as this on just one computer algebra package would not be wise. Also, my colleagues and I can still find faults in most of these packages, in much the same way as no spell-checker is perfect in a word processor. The upshot of this is that I have included much of the algebraic manipulation in the solved problems. In mechanics, there is often an obvious point when all the mech-

anics has been done and the rest is calculus or algebra. Nevertheless, in order to get an answer, the solving of the equations is necessary. The reader thus has the choice of checking (following) the manipulations by hand or inputting the equations into the chosen package and executing the appropriate menu-driven instructions.

This *Work Out* assumes that the student is familiar with mechanics as presented to 17 to 19 year-olds. If you have zero knowledge or have had a very bad educational experience with regard to mechanics, my *Guide to Mechanics* (with Roger Whitworth) should cover the preliminary material. There is also some overlap (no bad thing, in my view). The organisation of material in a book such as this is never easy, and for mechanics this seems particularly so. Chapter 1 is a reprise of preliminary notions in mechanics. It is this chapter that should tell you if you need to see the *Guide* first. Chapters 2–10 cover material usually found in the first year of an undergraduate course in mechanics for mathematicians or physicists. It should also cover *all* the mechanics commonly found in engineering and technology courses. I have decided to include Lagrange's equations and Euler's equation (tops and gyroscopes) for completeness. This material usually occupies the second year of mathematics degrees and is absent from engineering degrees; however, it is a crucial precursor for theoretical physicists who wish to study quantum mechanics. It is also, incidentally, a wonderfully systematic treatment of mechanics that often appears easy to those who complain at length of the *ad hoc* nature of earlier mechanics. Lagrange's equations almost seem like pure mathematics. Finally, Chapter 12 is a brief introduction to non-linear mechanics, which is a very fashionable area to study these days. Everyone seems to have heard of 'chaos', although few, I think, understand it.

In summary, therefore, you can expect to find the solved problems here representative of those in most undergraduate mechanics courses. In line with other *Work Out* texts, the material in this book can certainly be used in any order. There is very little cross-referencing, except perhaps for information. The 'fact sheet' at the beginning of each chapter is a brief summary of relevant formulae and results which is best used as a reference.

Phil Dyke
December 1993

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Finally, special thanks to my wife Heather, who spent long hours word processing when there was so much else to do.