Essential Mathematics

ESSENTIAL MATHEMATICS

**A Refresher Course for Business
and Social Studies**

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and
Emmanuel Thanassoulis

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Preface

This book originated in our experience of teaching preliminary mathematics courses for undergraduates about to embark on degrees in management, accounting and related disciplines, and for postgraduates about to start on MBA courses. Many of these students were nervous at the prospect of having to study a numerate discipline, whether called statistics, quantitative methods or whatever, as part of their intended course. For some, earlier experiences of studying mathematics had not been happy, and had left a legacy of confusion. Others had been perfectly competent mathematicians, but had not studied the subject for a number of years, and were anxious that they had become rusty. There was also a fear that certain topics which had not formed part of their previous experience – such as calculus – might be an essential prerequisite to their future course of study.

If you feel that the descriptions above apply to you, then this book is addressed to your needs. It is aimed at adults – so we hope you will find the tone not too reminiscent of a school-level text. It is designed for people who are primarily interested in professional subjects – business, management, accounting and so on – so there are practical examples drawn from those areas to illustrate and reinforce the mathematical principles. It is intended in the main for those working on their own; numerous worked examples, and practice exercises with full solutions, are therefore included.

The material of Chapters 1–4 is roughly that part of the GCSE/O-level syllabus which could be described as arithmetic and algebra. For most readers, therefore, we envisage that much of this material will fall into the category of revision of things you once knew, rather than totally new material. Chapters 5–7 cover calculus, solving practical problems with mathematics, and elementary statistics, which are not always included in GCSE-level

mathematics courses; in those chapters, we have imagined a reader to whom the concepts are completely new.

There are two points which we would like to make concerning the way you use this book. One is that no one ever learned mathematics by reading a book; the learning comes about through doing – through practising the techniques which have been described. So make sure that you do the exercises as you go along, and that if you disagree with one of the answers provided, you study the solution and ensure that you understand where you went astray before carrying on to the next section. Mathematics is to a large extent a linear subject – that is, one topic builds on another, so if you are confused about arithmetical operations you have little chance of getting to grips with algebra, and if algebra poses problems your grasp of calculus will probably be hazy. Thus the first five chapters of the book need to be read in sequence. If you find that you feel very comfortable with some of the material, you may wish to skim fairly quickly through the relevant text; we do, however, strongly recommend you to work through the exercises to verify whether you were right to feel confident!

The second point is chiefly a matter of attitude. Many people believe that mathematics is a rather mysterious and elusive subject, composed of abstractions, and accessible only to a minority of the population possessed of extraordinary minds. While it is true that there *are* areas of advanced mathematics to which this description would apply, it is also the case that elementary mathematics is extremely practical stuff, rooted in common sense, and useful for dealing with everyday problems in an efficient manner. That students studying the subject often do not have the opportunity to discover this is a sad reflection on those of us responsible for introducing them to mathematics. Our aim in writing this book has been to de-mystify the processes of elementary mathematics, and show just how useful they are in the context of business and management problems. If you, the reader, feel by the end of the book that it is really all rather simple stuff, we will have succeeded!

We are grateful to John Thanassoulis for checking the solutions to most of the exercises, and for drawing many of the diagrams. We would like to thank colleagues at Warwick Business School who have commented on earlier versions of some of this material produced for the Warwick Distance Learning MBA. We are also indebted to numerous students who, in attending the Preliminary

Mathematics Courses mentioned above, have unwittingly acted as guinea-pigs for our ideas. As always, any errors which remain are entirely our responsibility, and we will be happy to be informed of them.

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