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A-Level

B. Haines
and
R. Haines

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MACMILLAN
EDUCATION

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To our Parents

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The University of London Entrance and School Examinations Council accepts no responsibility whatsoever for the accuracy or method in the answers given in this book to actual questions set by the London Board.

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The Associated Examining Board, the University of Oxford Delegacy of Local Examinations, the Northern Ireland Schools Examination Council and the Scottish Examination Board wish to point out that worked examples included in the text are entirely the responsibility of the authors and have neither been provided nor approved by the Board.

Examination Boards for A-Level

Syllabuses and past examination papers can be obtained from:

The Associated Examining Board (AEB)
Stag Hill House Guildford Surrey GU2 5XJ

University of Cambridge Local Examinations Syndicate (UCLES)
Syndicate Buildings Hills Road Cambridge CB1 2EU

Joint Matriculation Board (JMB)
78 Park Road Altrincham Ches. WA14 5QQ

University of London Schools Examinations Board (L)
University of London Publications Office 52 Gordon Square London WC1E 6EE

University of Oxford (OLE)
Delegacy of Local Examinations Ewert Place Summertown Oxford OX2 7BZ

Oxford and Cambridge Schools Examination Board (O&C)
10 Trumpington Street Cambridge BC2 1QB

Scottish Examination Board (SEB)
Robert Gibson & Sons (Glasgow) Ltd 17 Fitzroy Place Glasgow G3 7SF

Southern Universities' Joint Board
Cotham Road Bristol BS6 6DD

(SUJB)

Welsh Joint Education Committee
245 Western Avenue Cardiff CF5 2YX

(WJEC)

Northern Ireland Schools Examination Council
Examinations Office Beechill House Beechill Road Belfast BT8 4RS

(NISEC)

Introduction

This Work Out is not just another text book. It is based on the 'A' level Common Core Mathematics syllabus and has been designed to help students to obtain the best possible grades in their 'A' level examination. In addition it has been written to provide a bridge for students, especially BTEC students, wishing to go into higher education. Students entering higher education often find they are unfamiliar with some 'A' level topics and lack sufficient practice in topics which the lecturers assume to be known.

Each chapter in the book starts with a brief list of formulae and is followed by many 'A' level type questions, each with a complete solution. At the end of each chapter more 'A' level questions are set as exercises with the important stages in the working of each one being given to facilitate easy reference.

How to Use the Book

- (a) By repeated use and practice, endeavour to become familiar with the frequently used formulae listed in the fact sheets.
- (b) Practice in answering examination questions is important. Open the book at a definite topic, choose a question and cover up the solution until you have tried to do it by yourself. If you get really stuck, your mind will be receptive when you uncover the solution.
- (c) When answering multiple-choice questions, work through your solution in the same way as you would an ordinary question. If you are unable to answer the question, substituting the suggested answers into the question can often help to eliminate some solutions. Only guess an answer as a last resort, and not even then if the marking scheme penalizes wrong answers.

Revision

- (a) Your educational establishment should be able to supply you with a syllabus and typical examination papers. Failing that, you should write to the secretary of the board whose examination you plan to take. A list of addresses is given on pages vii and viii.
- (b) Use the book to revise topics you have already covered, before trying complete papers.
- (c) Frequent reference to the formula booklet in examinations wastes precious time and increases the risk of losing your train of thought in the middle of a solution, while, as often as not, the formula you seek is not listed. Familiarize yourself with the contents of your formula sheet before the examination!
- (d) Development of an examination technique is very important and may take months to evolve. Well before the examination, may be before you finish the syllabus, choose a convenient time and do a paper 'to time' under examination conditions (quiet!). You may well be appalled at how badly you do. Go through the paper again as soon as possible, referring to similar questions in this book. At

this time your mind will be receptive to ways of overcoming the difficulties you have encountered. If you are able to do one paper 'to time' each fortnight for two or three months you will develop a sixth sense which enables you to spot 'easy' questions.

(e) Get into the habit of writing solutions tidily *first time*. So many students are content with scruffy solutions during the months leading up to the examination. Perhaps they expect a fairy to wave a wand and change their untidiness into a perfect script on the day! Unfortunately there is a shortage of such fairies, so get into the 'neat first time' habit.

The Examination

(a) Do the 'easy' questions at the start of the examination; this boosts your confidence.

(b) If you get stuck on a question and cannot see an alternative approach, cut your losses and go to another question. Examinations have been failed by good candidates spending too long stuck on one or two questions.

(c) If you find you are running out of time then look for questions which give 'part-way' answers, usually after asking bookwork. Quite often the problem part of the question is a straightforward application of the bookwork, is easy to do and is rewarding in marks. Do it, even if you cannot do the bookwork.

(d) Never cross out. You may be crossing out marks.

(e) Never walk out of an examination. Re-read the questions right through; even if you cannot do the first parts of the questions there will be some parts of some questions you can try.

(f) Good luck!